Web Appendix

Decomposing Promotional Effects with a Dynamic Structural Model of Flexible Consumption

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Web Appendix A. Reduce a Infinite-Horizon Problem to a Finite-Horizon Problem

Our objective is to reduce the infinite-horizon problem in Equation 1 to a finite-horizon problem. First, we can replace the purchase and the inventory in period t, x_t and I_{t-1}, with the consumption at current and all future periods, i.e., $x_t = \sum_{s=t}^{\infty} y_{t,s}$ and $I_t = \sum_{s=t}^{\infty} y_{t-1,s}$ The subscript "t" in $y_{t,s}$ denotes the time of purchase, and "s" the time of consumption, and "t-1" in $y_{t-1,s}$ indicates that it is quantity inherited from inventory at period t-1. Then we can rewrite the infinite horizon planning problem into

$$\sup_{\{\}} E_t u(y_{t-1,t} + y_{t,t}) - \lambda \cdot p_t' \left(\sum_{s=t}^{\infty} y_{t,s}\right) - c \cdot \left[\sum_{j=1}^{J} \sum_{s=t+1}^{\infty} (y_{t-1,s,j} + y_{t,s,j})\right] + \sum_{s=t+1}^{\infty} \gamma^{s-t} E_t \qquad \text{(B.1)}$$

$$\{u(y_{t-1,s} + y_{t,s} + \ldots + y_{s,s}) - \lambda \cdot (p_s^0)' \left(\sum_{u=s}^{\infty} y_{s,u}\right) - c \cdot \left[\sum_{j=1}^{J} \sum_{u=s+1}^{\infty} (y_{t-1,u,j} + y_{t,u,j} + \ldots + y_{s,u,j})\right] \| \sigma_t\}$$

$$\text{s.t. } I_{t-1} = \sum_{s=t}^{\infty} y_{t-1,s}$$

$$y_{s,u,k} \geq 0,\ u \geq s,\ k = 1,\ldots,J$$

Since $\overline{p}_j - p_{tj} < b < \infty$, for all t and j, i.e., the difference between the highest possible price in data, $\overline{p}_j$, and the observed price of product j, p_{tj}, is finite, there exists a finite time period, T_1, such that households will not purchase at time t for the consumption of time u, for all $u > T_1$. This is under the condition that the cost from purchasing the product today and stockpiling for consumption at period T_1 is greater than the cost of having to buying the product at period T_1 and consume it at the same period (even at the highest price), i.e.,

$$\lambda \cdot p_{tj} + c \cdot \sum_{u=t}^{T_1} \gamma^{u-t} > \lambda \cdot (\gamma^{T_1-t}\ \bar{p}_j)$$

$$\Rightarrow T_1 - t < \frac{\log((\frac{c}{\lambda} + p_{tj})/\bar{p}_j)}{\log\gamma} \qquad \text{(B.2)}$$

In this case, $y_{t,u,j} = 0$, for all $u > T_1$. That is, no household will purchase product j at t and hold it as inventory for the consumption after period T_1.

Now let us consider the endogenous variables $\{y_{t-1,t}, y_{t-1,t+1}, \ldots\}$, i.e., the inventory inherited from period t-1 which will be used for future consumption. By assumption, marginal utility (i.e., $\frac{\partial E_t u(y_s)}{\partial y_j} | y_{s=0} = \alpha \cdot \Psi_t' A_j'$) is finite at zero consumption level. Thus, there exists a T_2 such that

$$\alpha \cdot \Psi' A_j \cdot \gamma^{T_2-t} < c \cdot \sum_{u=t}^{T_2} \gamma^{u-t}$$

$$\Rightarrow T_2 - t < [\log(c) - \log(\alpha \cdot \Psi' A_j \cdot (1-\gamma) + c)]/\log\gamma \qquad \text{(B.3)}$$

This implies that a household will not stockpile at t for the consumption after period T_2 due to the existence of its inventory cost and the discount rate.

Let $T = \max\{T_1, T_2\}$. We can write a finite horizon problem equivalent to the infinite horizon problem in Equation B.1

$$\sup_{\{\}} E_t u(y_{t-1,t} + y_{t,t}) - \lambda \cdot p_t' (\sum_{s=t}^{T} y_{t,s}) - c \cdot [\sum_{j=1}^{J} \sum_{s=t+1}^{T} (y_{t-1,s,j} + y_{t,s,j})] + \sum_{s=t+1}^{T} \gamma^{s-t} E_t \qquad \text{(B.4)}$$

$$\{u(y_{t-1,s} + y_{t,s} + \ldots + y_{s,s}) - \lambda \cdot (p_s^0)' (\sum_{u=s}^{T} y_{s,u}) - c \cdot [\sum_{j=1}^{J} \sum_{u=s+1}^{T} (y_{t-1,u,j} + y_{t,u,j} + \ldots + y_{s,u,j})] | \sigma_t\}$$

$$\text{s.t. } I_{t-1} = \sum_{s=t}^{T} y_{t-1,s}$$

$$y_{s,u,k} \geq 0,\ u \geq s,\ k = 1,\ldots,J.$$

Solutions of $y_{t-1,s}$ and $y_{t,s}$, for all $s \geq t$, in Equation B.4 are equivalent to solutions in Equation B.1. Therefore, our infinite horizon problem has been reduced to a finite horizon planning problem.

Web Appendix B. Model Estimation (Simulated Method of Moments)

First, we examine how a household decides whether to buy, which product to buy, how much to consume, and how much to buy. To simplify the analysis, suppose that the household does not hold inventory in week t. The utility function in Equation 2 implies that, after observing p_t, a household will choose at most one product, j*, such that, for all k = 1, …, J,:

$$\frac{MU_{j*}(0)}{\lambda p_{t,j*}} = \frac{\alpha \cdot \Psi' A_{j*}}{\lambda p_{t,j*}} \geq \frac{\alpha \cdot \Psi' A_k}{\lambda p_{t,k}} = \frac{MU_k(0)}{\lambda p_{t,k}}$$
$$\Rightarrow \frac{\Psi' A_{j*}}{p_{t,j*}} \geq \frac{\Psi' A_k}{p_{t,k}} \qquad \text{(C.1)}$$

where $MU_{j*}(0)$ is the marginal utility level with respect to j* at $y_k = 0$, $\forall k = 1,\ldots,J$. Corner solution exists when $\max_{\{k\}}\{\frac{\alpha \cdot \Psi' A_k}{\lambda p_{t,k}}\} < 1$, for all k. In such a case we have x_t = 0. This occurs when the household finds current prices too high to purchase the product category at week t.

With the expected price p_t^0 and the utility function as specified in Equation 2, the household expects to choose at most one product j^0 in any future period s, such that, for all k,

$$\frac{MU_{j^0}(0)}{\lambda p^0_{t,j^0}} = \frac{\alpha \cdot \Psi' A_{j^0}}{\lambda p^0_{t,j^0}} \geq \frac{\alpha \cdot \Psi' A_k}{\lambda p^0_{t,k}} = \frac{MU_k(0)}{\lambda p^0_{t,k}}$$
$$\Rightarrow \frac{\Psi' A_{j^0}}{p^0_{t,j^0}} \geq \frac{\Psi' A_k}{p^0_{t,k}} \qquad \text{(C.2)}$$

Again, when $\max_{\{k\}}\{\frac{\alpha \cdot \Psi' A_k}{\lambda p^0_{t,k}}\} < 1$, for all k, the expected purchase in period s will be zero. This occurs when the household does not normally purchase or consume the category, and will only purchase when there is a big price promotion.

As the above discussion implies, the household will purchase in week t and hold it for consumption in week s, where s>t, only if the following two conditions are satisfied:

(i) $$\gamma^{s-t} \cdot MU_{j*}(0) - \lambda p_{t,j*} - c \cdot \sum_{u=0}^{s-t} \gamma^u \geq 0$$

and

(ii) $$\gamma^{s-t} \cdot MU_{j*}(0) - \lambda p_{t,j*} - c \cdot \sum_{u=0}^{s-t} \gamma^u \geq \gamma^{s-t} \cdot [MU_{j^0}(0) - \lambda p^0_{t,j^0}]$$

Condition i ensures that discounted consumption utility in week s net of purchasing costs in week t and discounted inventory cost is non-negative. Condition ii ensures that it is worthwhile to buy now and hold inventory until week s.

Suppose the above two conditions are satisfied, i.e., the household purchases in week t and holds it for consumption in week s. The optimal purchase quantity then satisfies the third condition that is derived from the first-order condition:

(iii) $$\gamma^{s-t} \cdot MU(y_{t,s,j*}) - \lambda p_{t,j*} - c \cdot \sum_{u=0}^{s-t} \gamma^u = 0$$

where $y_{t,s,j*}$ is the optimal quantity purchased in week t for the consumption in week s. The optimal level of x_t is equal to the sum of $y_{t,s,j*}$, s=t,…,T, in the j*-th row and zero elsewhere. Thus, the optimal proportion of consumption $\delta^*_s = y_{t,s,j*} / x_{t,j*}$.

When the household holds positive inventory in week t, solution $y_{t,s,j*}$ for all s cannot be solved separately as in Conditions i to iii, since the household has to additionally consider the benefit and cost of consuming the inventory vs. buying in current week. However, basic principles of the solution concept discussed above still apply. In our algorithm, we directly solve the non-linear constrained optimization problem in Equation 5, given parameters θ and inventory level I_{t-1}.

Based on the above discussions, an appropriate technique is to use the Method of Moments by matching the expected quantity purchased obtained from the maximization problem with the observed purchases. The estimation procedure involves a nested algorithm for estimating parameters θ: an "inner" algorithm computes a simulated quantity purchased to solve the problem in Equation 5 for each trial value of θ, and an "outer" algorithm searches for the value of θ that minimizes a criterion function. The inner algorithm is repeated every time when θ is updated by the outer algorithm.

Procedure for the inner algorithm is the following: Given the conditional distribution function of the random variables ε (here, ε includes all stochastic components in the utility function, the standard normal variable that relates to $I_{h,0}$, and the binomial variable that determines which type h.), $F(\cdot|X_t)$, where X_t represents all explanatory variables including marketing variables such as prices, product features and displays, as well as demographic variables like household size, income level, residence type, female employment status, we can solve the expected values of $\{x_{t,\ldots,}x_{t+T;}\ \delta_{t,\ldots,}\delta_{t+T}\}$ from Equation 5, i.e.,:

$$\{x_t^*,\ldots x_{t+T}^*;\delta_t^*,\ldots,\delta_{t+T}^*\} = \int_{\{x_t,\ldots x_{t+T};\delta_t,\ldots\delta_{t+T}\}} \text{argmax } \{\text{problem in Equation 5 } F(d\varepsilon|X_t)\} \qquad \text{(C.3)}$$

However, x_t* in Equation C.3 is non-tractable because of the non-negativity constraints. Instead, we use a simulation method: for each household h and period t, we draw

$\tilde{\varepsilon}_{ht,1},\ldots,\tilde{\varepsilon}_{ht,ns}$ from the distribution function $F(\cdot|X_t)$, where ns is the number of simulation draws. Conditional on each simulation draw $\tilde{\varepsilon}_{ht,s}$, we solve a non-linear constrained optimization problem for the optimal quantity purchased levels at time t, $\tilde{x}(X_{ht};\theta;\tilde{\varepsilon}_{ht,s})$, using a derivative search procedure called the Sequential Quadratic Programming. In this method variables are updated in a series of iterations beginning with a starting value that satisfies the constraints in Equation 5. If $\tilde{x}_n$ is the current value at iteration n, then its successor is $\tilde{x}_{n+1} = \tilde{x}_n + \tau \cdot \varsigma$, where ς is a direction vector, and τ a scalar step length. The procedure is repeated for every simulation draw and this generates a simulated dataset $\tilde{x}(X_{ht};\ \theta) = (1/ns) \cdot \sum_{s=1}^{ns} \tilde{x}(X_{ht};\theta;\tilde{\varepsilon}_{ht,s})$. When the utility function is concave and its Jacobian and Hessian matrices can be written down in analytical forms, convergence of $\tilde{x}$ is fast.

The outer algorithm searches for the estimator θ. We make a major identification assumption that there are no unobserved characteristics in the model. Therefore, there is no endogeneity issue for marketing variables such as prices. Although this assumption can be challenged, it solves the data problem as good instruments for weekly prices are not available. Under the identification assumption, this yields a moment condition:

$$E[x_{ht} - \tilde{x}(X_{ht};\ \theta_0)] = 0 \qquad \text{(C.4)}$$

where θ_0 is the true parameters. The estimator θ_n is obtained by the following non-linear least-square estimator

$$\theta_n = \text{argmin}_{\{\theta \in \Theta\}} Q_n(\theta) = \text{argmin}_{\{\theta \in \Theta\}} \frac{1}{H \times T_h} \sum_{h=1}^{H} \sum_{t=1}^{T_h} [x_{ht} - \tilde{x}(X_{ht};\ \theta)]^2 \qquad \text{(C.5)}$$

We use the Nelder-Meade (1965) nonderivative simplex method to search for θ_n. Estimators based on this moment condition are called the Simulated Method of Moment (SMM) estimators (Pakes (1986), Pakes and Pollard (1989), McFadden (1989)). One major advantage of using the SMM is that ns can be finite (even when ns = 1) and we still obtain consistent estimators. This helps to reduce the computational burden in model estimation. Our methodology here is very similar to Chan (2006). However, his model is to estimate multiple-product, multiple-unit purchase decisions, while our model is to estimate the purchase and consumption decisions over multiple periods.

Web Appendix C. Identification of Different Types of Consumption Behavior

Depending on the exogenous (1) preferences for the attributes of different products, (2) inventory cost, and (3) the slope coefficient α, a price promotion will have different effects on different households. A major identification issue in our estimation is that we, as researchers, do not observe consumption and inventory of households in the data. We only observe whether a household makes a purchase and if so which product it buys and the quantity it purchases in each period. Still, we can identify the parameters of a household's utility function from the observed variations in its purchasing pattern over time: brand switching patterns of households over time help to identify the differences in product preferences of households, and variations in purchase quantity and time-intervals between purchases help to identify the inventory cost and consumption rate changes. For example, suppose that there are two households, A and B, who buy one unit of the product in each period at the same price. Suppose the price is cut by 10 percent in the current period, and both A and B increase their purchase from 1 to 2 units. If household A comes back to purchase 1 unit again in the next period, but household B does not make a purchase until period 3, we can infer that A increases its consumption and does not stockpile in the current period, while household B does the opposite. In such a case household A has a flexible consumption rate but a higher inventory cost than household B. Suppose there is another household, C, which buys 4 units of the product during the promotion, and only comes back to market in period 3. Then we can infer that household C may have a more flexible consumption rate than household B but a lower inventory cost than household A. Hence, the parameters are identifiable if there are enough variations on purchases quantity and inter-purchase time-intervals in responses to prices.

Given these parameter values, effects of stockpiling, brand-switching and consumption increase due to temporary price promotions can then be identified. For example, when there is a price promotion household A will show a larger consumption effect but a smaller stockpiling effect than household B does, while household C will show a larger consumption as well as stockpiling effect.

Another identification issue arises from the fact that we do not observe households' price expectations, and the parameter ω in Equation 4 has to be estimated. Suppose in week t household h does not have any inventory. Assume that household h observes a price higher than its expected price p^0_{ht} and decides to buy one unit. If in week t+1 the price is the same as last week (i.e., $p_{t+1} = p_t$) and h decides to buy more. In this case one can infer that h's expected price $p^0_{h,t+1}$ must be adjusted upward, and ω is positive (this is because h would not purchase for stockpiling purpose in week t+1 if it was the case that $p^0_{h,t+1} = p^0_{ht} < p_{t+1} = p_t$).

In order to demonstrate that the model parameters can be identified, we conduct a simulation by fixing the parameters and stochastic components of the objective function in Equation 5 to generate the household purchasing and consumption data. Next, we estimate the model by using another set of randomly drawn stochastic numbers. The results are available from the authors upon request. Overall the estimated parameters are very close to the "true" ones, rendering support for the above arguments.

Web Appendix D. Some Summary Statistics of Tuna Products

Product alternative	Market Share (%)	Avg. Price ($/Unit)	Avg. Feature	Avg. Display
StarKist, Water, Light	36.32	.78	.14	.04
CKN, Water, Light	32.56	.81	.13	.05
StarKist, Oil, Light	13.00	.70	.16	.04
CKN, Oil, Light	10.05	.82	.12	.05
CTL, Water, Light	3.44	.65	.02	.04
3 Diamond, Water, Light	1.66	.62	.01[1]	.08
CTL, Oil, Light	1.24	.65	.03	.04
3 Diamond, Oil, Light	.71	.62	.01	.08
Other Brands	.54	1.14	.00	.00
CTL, Oil, White	.27	.64	.02	.11
StarKist, Water, White	.19	1.54	.00	.00
CTL, Water, White	.01	1.22	.00	.00

Web Appendix E. Comparison of True and Estimated Parameters from Simulated Data

	True Parameter	Estimated Parameter	Std. Error
Brand Preference	3.00	2.79	.12
Inventory Cost	.03	.12	.01
Power Term (α)	.50	.52	.003
Brand Preference Heterogeneity (σ)	1.00	.94	.04
Price Updating Parameter(ω)	N/A	.12	.56

Web Appendix F. Comparison of Decompositions of Promotional Effects from the True and Proposed Models

	True Model	Proposed Model
Total Effect	-9.38 (100%)	-8.42 (100%)
Consumption Effect	-5.39 (57%)	-4.50 (53%)
Stockpiling Effect	-3.99 (43%)	-3.92 (47%)

Note: The percentages relative to the total effect are in parentheses.

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