Web Appendix

Brand Portfolio Promotions

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Technical Details for Model Estimation

In estimating the model, we incorporate store heterogeneity only for the control variables. So, first we specify:

$$x^*_{jst} = (1, Time_{st}, Q^{'}_{st}, price_{jst}, feature_{jst}, display_{jst}, comp_{jst}, comf_{jst}, comd_{jst})'$$

$$x^{**}_{jst} = (\sum_m \lambda^m_{jst} \operatorname{cov}^m_{js} I^m_{jst})'$$

$$x^*_{st} = (x^*_{1st}, ..., x^*_{Jst}) \text{ and } x^{**}_{st} = (x^{**}_{1st}, ..., x^{**}_{Jst})$$

$$X^*_{js} = \begin{pmatrix} x^{*'}_{js2} - x^*_{s1}\phi_j \\ \vdots \\ x^{*'}_{jsT} - x^*_{sT-1}\phi_j \end{pmatrix} \text{ and } X^{**}_{js} = \begin{pmatrix} x^{**'}_{js2} - x^{**}_{s1}\phi_j \\ \vdots \\ x^{**'}_{jsT} - x^{**}_{sT-1}\phi_j \end{pmatrix}, \text{ where}$$

the distribution of heterogeneity for all control variables $\eta_s \sim N(\overline{\eta}, D_\eta)$. The MCMC algorithm (Gelfand and Smith 1990) is cycled through as follows:

1) Generate $\{\eta_s \mid s = 1, ..., S\}$, η_s is a $(J \times 11) \times 1$ vector of parameters associated with control variable for store s. We specify:

$$X^*_s = \begin{pmatrix} X^*_{1s} & & \\ & \ddots & \\ & & X^*_{Js} \end{pmatrix}$$

$$Y^*_s = \begin{pmatrix} y^*_{1s2} - y^*_{s1}\phi_1 \\ \vdots \\ y^*_{jst} - y^*_{st-1}\phi_j \\ \vdots \\ y^*_{JsT} - y^*_{sT-1}\phi_J \end{pmatrix}, \text{ where } y^*_{jst} = y_{jst} - x^{**}_{jst}\tau_{oj} - w_{jst}\gamma \text{ and } y^*_{st} = (y^*_{1st}, ..., y^*_{Jst}).$$

Then $\eta_s \sim MVN(a_s, A_s)$, where

$A_s = [X_s'^{*}(\Sigma^{-1} \otimes I_T)X_s^{*} + D_\eta^{-1}]^{-1}$ and $a_s = A_s[X_s'^{*}(\Sigma^{-1} \otimes I_T)Y_s^{*} + D_\eta^{-1}\overline{\eta}]$.

2) Generate τ_o, a vector of τ_{oj}, j=1,…,J and $\gamma' = (\gamma_h, \gamma_l, \gamma_p)$. X_s^{**} and Y_s^{**} are structured in the same manner as X_s^{*} and Y_s^{*}. We specify:

$$W_{js}^{*} = \begin{pmatrix} w_{j12}^{*'} - w_{11}^{*}\phi_j \\ \vdots \\ w_{jst}^{*'} - w_{st-1}^{*}\phi_j \\ \vdots \\ w_{jST}^{*'} - w_{ST-1}^{*}\phi_j \end{pmatrix}, \text{ where } w_{st}^{*} = (w_{1st}^{*}, ..., w_{Jst}^{*})$$

$$Z_s^{*} = \begin{pmatrix} X_{1s}^{**} & & & W_{1s}^{*} \\ & X_{2s}^{**} & & W_{2s}^{*} \\ & & \ddots & \vdots \\ & & X_{Js}^{**} & W_{Js}^{*} \end{pmatrix}$$

$$Y_s^{**} = \begin{pmatrix} y_{1s2}^{**} - y_{s1}^{**}\phi_1 \\ \vdots \\ y_{jst}^{**} - y_{st-1}^{**}\phi_j \\ \vdots \\ y_{JsT}^{**} - y_{sT-1}^{**}\phi_J \end{pmatrix}, \text{ where } y_{jst}^{**} = y_{jst} - x_{jst}^{**}\eta_{js} \text{ and } y_{st}^{**} = (y_{1st}^{**}, ..., y_{Jst}^{**})$$

Y^{**} and Z^{*} are constructed by stacking up Y_s^{**} and Z_s^{*}, respectively.

Then $\begin{pmatrix} \tau_o \\ \gamma \end{pmatrix} \sim MVN(b, B)$, where

$B = [Z'^{*}(\Sigma^{-1} \otimes I_{TxS})Z^{*} + \Lambda_\tau^{-1}]^{-1}$ and $b = B[Z'^{*}(\Sigma^{-1} \otimes I_{T\times S})Y^{**} + I_J\sigma_{\tau_o}^2\overline{\tau}_o]$. The prior for τ_o is specified as $\tau_o \sim N(\overline{\tau}_o, I_J\sigma_{\tau_o}^2)$.

3) Generate β a vector of β_j, j=1,…, J. Because of its nonlinearity, β are estimated using the Metropolis-Hasting algorithm with

$$p(\beta \mid Y, X^*, Z^*, \mu, \gamma) \propto l(\mu, \gamma, \beta, \Sigma \mid Y, X^*, Z^*) N(\beta_o, \Lambda_\beta) \text{ , where}$$

$$l(\mu, \gamma, \beta, \Sigma \mid Y, X^*, Z^*) \propto \left|\Sigma^{-1}\right|^{-T \times S/2}$$
$$\exp\left(-\frac{1}{2}(Y^* - X^*\eta - X^{**}\tau_o - W^*\gamma)'\Sigma^{-1} \otimes I_{T\times S}(Y^* - X^*\eta - X^{**}\tau_o - W^*\gamma)\right)$$

and $N(\beta_o, \Lambda_\beta)$ is the prior distribution for β. The μ is a column vector of linear parameters to be estimated, which includes η and τ_0.

4) Generate Σ

   $\Sigma \sim InvWishart(h, H)$, where

   $h = h_o + T$

   $H^{-1} = H_o^{-1} + (Y^* - X^*\eta - X^{**}\tau_o - W^*\gamma)'(Y^* - X^*\eta - X^{**}\tau_o - W^*\gamma)$

   The prior for Σ is $InvWishart(h_o, H_o)$.

5) Generate $\overline{\eta}$, a $(J \times 11) \times 1$ vector of means of store-level parameters associated with control variables.

   $$\overline{\eta} \sim MVN\left(\frac{\sum_s \eta_s}{S}, \frac{D_\eta}{S}\right).$$

6) Generate D_η, a matrix that captures variation of η_s across stores.

   $D_\eta \sim InvWishart(g, G^{-1})$, where

   $g = g_o + S$ and $G^{-1} = G_o^{-1} + \sum_s (\eta_s - \overline{\eta})'(\eta_s - \overline{\eta})$. The prior for D_η is

   $InvWishart(g_o, G_o^{-1})$.

7) Generate Φ, let $e_{st} = (e_{1st}, ..., e_{Jst})$, where $e_{jst} = y_{jst} - x'^{*}_{jst}\eta_{js} - x'^{**}_{jst}\tau_j - w'_{jst}\gamma$.

Then, $E_s = (e_{s1},...,e_{sT-1})'$ and E is constructed by stacking up E_s.

$\Phi' \sim MATN(\overline{\Phi}', (E'E)^{-1} \otimes \Omega) \times I(\Phi)$, where $\overline{\Phi}' = (E'E)^{-1} \times (\sum_s \sum_{t=2}^{n} e_{st-1} e'_{st})$ and

I(Φ) is an indicator function that takes the value of 1 when all roots of *Φ* lie in the unit circle and 0 otherwise. Following Chib and Greenberg (1995), we simulate the distribution of *Φ* by specifying $E^* = \overline{\Phi}' + PTQ'$, where T is a k x k matrix that consists of independent standard normal variables, and P and Q satisfy the Choleski factorizations $PP' = (E'E)^{-1}$ and $QQ' = \Omega$, respectively. If all the roots of E^* are less than unity (the stationary condition) then E^* is accepted; otherwise, another T is drawn and the process repeats.

An Illustration of Optimal Exposure Allocation for a BPP Program

The following optimal exposure allocation exercise is conducted across multiple BPPs. Our goal is to develop a promotion exposure calendar for a portfolio of brands. For the purpose of illustration, we restrict our attention to the ten brands included in this paper and on BPP5, BPP6 and BPP7 (i.e., a snap shot from a BPP program). The actual relative exposure for brands A-J is reported in Table 1A. For example, for BPP5 the relative exposure for Brand A was 24% and Brand I was not included. For a given BPP, relative levels of exposure for all brands are summed to 100.

In the following analyses, we quantify the profitability of the BPP exposure calendar actually used by the company (i.e., the status-quo scenario) and assess ways in which it could be improved. The objective function is to maximize expected total incremental profit for the ten brands across the three BPPs subject to a space (i.e., the total exposure) constraint. To make an objective and managerially relevant comparison we assume that (1) each brand has an opportunity for being present in each BPP; (2) the total exposure in each BPP remains unchanged to control the total costs for the current and new allocations to be the same; and (3) new exposure for each brand remains within two standard deviations of its average exposure across the fourteen BPPs to assure that the new allocation is actually plausible (e.g., it may be more difficult to include a certain brand in recipes.).

Formally, our goal is as follows:

$$\max_{I,\omega} \sum_s \int g(I,\omega,\Omega_s)\, p(\Omega_s \mid Y, X)\, d\Omega \text{, where}$$

$$g(I,\omega,X_s,\Omega_s) = \sum_t \sum_j \sum_m \pi(I_j^m, \omega_j^m, X_{st}, \Omega_s)$$

$$= \sum_t \sum_j \sum_m \left(sale(I_j^m, \omega_j^m, X_{st}, \Omega_s) - sale(X_{st}, \Omega_s) \right) m_j$$

$$\text{s.t. } \sum_j I_j^m \omega_j^m = K_m, \forall m \text{ and } \omega_j^m < 1.96 * \sigma_{w_j}, \forall j$$

I_j^m is equal to1 if brand j is featured and 0 otherwise, ω_j^m is the exposure level for brand j in the mth BPP, and K_m is the sum exposure across ten brands for the mth BBP. The term $\pi(I_j^m, \omega_j^m, X_{st}, \Omega_s)$ represents the expected incremental profit for store s in week t generated from the mth BPP for brand j, $sale(I_j^m, \omega_j^m, X_{st}, \Omega_s)$ denotes the estimated sales of brand j in stores s in week t generated from the mth BPP, and $sale(X_{st}, \Omega_s)$ is the estimated baseline sales in store s in week t for brand j without its exposure in the mth BPP. The profit margin[1] for brand j is denoted by m_j and the standard deviation of brand j's exposure across the 14 BPPs by σ_{ω_j}. The $p(\Omega_s \mid Y, X)$ is the posterior distribution of all relevant parameters given our independent variables (X) and sales data (Y).

We use store-level parameter estimates described earlier for the purpose of this exercise. We begin by computing expected incremental profit, as compared to baseline sales, from BPPs at the current levels of brand exposure. This serves as the status quo scenario. Then we searched for the optimal exposure of different brands across the three

[1] A profit margin for each brand was provided by the company and asked to be kept confidential.

BPPs. The percent profit gain because of the exposure reallocation over the status-quo scenario thus demonstrates that our model can help the company further improve the effectiveness of its BPP program.

Given that we are dealing with a multidimensional problem of optimal BPP allocation across brands and BPPs, a brute force search for optimal exposure from all possible combinations was infeasible. The optimization difficulty is further exacerbated by our optimization procedure that computes expected incremental profit based on draws from the posterior distribution. To get around the problem, we use a greedy search technique similar to that used by Krieger and Green (1985)[2]. Specifically, instead of using the objective function directly, we treat our optimization problem as sequentially assigning each exposure unit to brand that provides the greatest increase in expected incremental profit.

[2] We do not claim that we can find a global optimum using such a heuristic search algorithm.

Table WA1: Results from Model-Based Optimal Space Allocation Exercise

	Relative Exposure in each BPP										% Profit Gain from Reallocation
	Brand A	**Brand B**	**Brand C**	**Brand D**	**Brand E**	**Brand F**	**Brand G**	**Brand H**	**Brand I**	**Brand J**	
Current plan (Actual Exposure)											
BPP 5	24	9	8	27	8	9	4	7	0	3	
BPP 6	24	9	7	16	7	8	9	7	7	7	
BPP 7	44	27	4	5	0	11	3	1	4	1	
Overall	30	15	6	16	5	9	5	5	4	4	n/a
Model-based plan (Optimal exposure)											
BPP 5	45	14	1	1	15	1	1	12	10	1	
BPP 6	0	24	14	1	13	1	8	10	9	21	
BPP 7	57	10	1	1	1	1	1	15	13	1	
Overall	34	16	5	1	10	1	3	12	11	7	5.32%