APPENDICES

A. DERIVATION OF THE ANNUALIZATION COEFFICIENT

Let us assume that a product that lasts for q years has a price of $1. Thus, a product originally produced at t = 0, will be replaced at times t = q, 2q, 3q…….If consumer h's discount rate is $r^h > 0$, then the present value, PV, of this stream of payments is:

(1) $$PV = 1 + \frac{1}{(1+r^h)^q} + \frac{1}{(1+r^h)^{2q}} + \frac{1}{(1+r^h)^{3q}} + ...\infty.$$

Since $\frac{1}{(1+r^h)^q} < 1,$ it follows that:

(2) $$PV = \frac{1}{1-(1+r^h)^{-q}}.$$

Now, let ρ^h be a series of payments that the individual h makes *every* period, i.e., t = 0, 1, 2…, such that the present value of these payments equals PV. Then,

(3) $$\rho^h + \frac{\rho^h}{(1+r^h)} + \frac{\rho^h}{(1+r^h)^2} + ... = \frac{1}{1-(1+r^h)^{-q}},$$

which implies,

(4) $$\rho^h \left[\frac{1}{1-(1+r^h)^{-1}}\right] = \frac{1}{1-(1+r^h)^{-q}}.$$

Finally,

(5) $$\rho^h = \frac{r^h}{1+r^h} . \frac{1}{1-(1+r^h)^{-q}}.$$

Intuitively, this implies that an individual with a discount rate of r^h and who intends to replace the product at intervals of q years is indifferent between paying $1 every q years or paying ρ^h every year.

B. MARKET AND VEHICLE CHARACTERISTICS

TABLE B-1: Market Characteristics

Model	Market Share (%)	Lease (%)	Finance (%)	Price ($)
Acura 3.2TL	27.7	16.5	44.1	29,134
Chrysler 300M	7.3	24.2	26.9	29,901
BMW 323i	6.3	29.6	30.7	32,979
Audi A4	3.8	27.2	24.9	30,275
Oldsmobile Aurora	1.2	21.5	24.0	32,487
Toyota Avalon	20.3	16.7	27.7	28,970
Mercedes Benz C2	4.8	38.6	22.0	32,741
Cadillac Catera	2.5	50.3	8.3	31,050
Lincoln LS	4.6	34.5	11.1	35,065
Mitsubishi Diamante	1.8	12.8	70.9	29,900
Lexus ES300	4.5	26.7	26.1	33,357
Infiniti I30	3.2	36.9	16.7	29,462
Mazda Millenia	1.4	20.1	31.4	27,339
Volvo S40	6.2	11.8	33.1	26,164
Volvo S70	4.4	15.4	33.7	30,065

Table B-2: Vehicle Characteristics

						Average Annual Maintenance Costs ($)**		
Model	Miles Per Gallon (MPG)*	Horse-power	Cylinders	Displacement†	Drivetype††	Buy	Lease 36	Lease 48
Acura 3.2TL	23	225	6	3.2	FWD	1036	532	674
Chrysler 300M	21	253	6	2.5	FWD	1167	591	710
BMW 323i	22	170	6	2.5	FWD	576	0***	119
Audi A4	24	190	6	2.8	AWD	637	130	242
Oldsmobile Aurora	20	250	6	3.5	FWD	914	401	487
Toyota Avalon	24	200	6	3	FWD	879	513	623
Mercedes Benz C2	24	185	4	2.3	FWD	814	186	297
Cadillac Catera	20	200	6	3	FWD	1047	521	636
Lincoln LS	20	252	8	3.9	FWD	942	191	352
Mitsubishi Diamante	20	210	6	3.5	FWD	1070	532	768
Lexus ES300	22	210	6	3	FWD	1185	725	800
Infiniti I30	23	190	8	3	FWD	940	538	559
Mazda Millenia	22	170	6	2.5	FWD	780	336	450
Volvo S40	24	160	4	1.9	FWD	496	98	185
Volvo S70	23	162	5	2.4	FWD	514	100	190

*Fuel Efficiency (FE) = 1/MPG

**Source: True Cost to Own data from www.edmunds.com

*** For some vehicles, maintenance and repair expenses are zero as they are covered by the contract itself.

† Displacement is the volume displaced while the pistons move.

†† Drivetype refers to the number of wheels receiving power from the drive shaft.

C. DERIVATION OF THE STRUCTURAL PARAMETERS

C-1. ANNUALIZATION COEFFICIENT

Consider the indirect utility function:

(6) $$\hat{V}_{ij}^{h} = \alpha_{0i}^{h} + \alpha^{h'}[X_{i}^{h}] + y^{h} - \rho^{h} NP_{ij} - M_{ij} + \mu_{j}^{h} - \theta_{0}^{h}\pi_{ij} + \kappa^{h}.$$

Now, the marginal rate of substitution between the net price (NP) and maintenance costs (M) is given by:

(7) $$\frac{d\hat{V}_{ij}^{h} \big/ dNP_{ij}}{d\hat{V}_{ij}^{h} \big/ dM_{ij}} = \frac{dM_{ij}}{dNP_{ij}} = -\rho^{h},$$

which is simply the discount factor, which in turn is a function of the discount rate, r^h.

Next, consider a general form of the above indirect utility function, where no restrictions are imposed on the parameters. In other words, the parameter associated with M_{ij} is no longer restricted to 1. We then re-write the IUF as:

(8) $$\hat{V}_{ij}^{h} = \alpha_{0i}^{h} + \alpha^{h'}[X_{i}^{h}] + y^{h} + \beta_{1}^{h} NP_{ij} + \beta_{2}^{h} M_{ij} + \beta_{j}^{h} + \beta_{3}^{h}\pi_{ij} + \kappa^{h}.$$

In this case, the marginal rate of substitution is given by:

(9) $$\frac{dM_{ij}}{dNP_{ij}} = -\frac{\beta_{1}^{h}}{\beta_{2}^{h}},$$

This implies that:

(10) $$\rho^{h} = \frac{\beta_{1}^{h}}{\beta_{2}^{h}}.$$

Intuitively, Equation 10 represents the change in maintenance cost which keeps utility constant for a one dollar change in the net price.

C-2. ANNUAL DRIVING DISTANCE

We can re-write the indirect utility function in Equation 6 as:

(11)

$$\hat{V}_{ij}^{h} = \begin{cases} \alpha_{0i}^{h} + \alpha^{h'}[X_i^h] + \mu_j^h + y^h - \rho^h NP_{ij} - M_{ij} - \theta_0^h fe_i + \kappa^h & \text{if } \theta_0^h \leq 12,000 \\ \alpha_{0i}^{h} + \alpha^{h'}[X_i^h] + \mu_j^h + y^h - \rho^h NP_{ij} - M_{ij} - 12000 fe_i - (\theta_0^h - 12000)(fe_i + \omega_{ij}) + \kappa^h, & \text{if } \theta_0^h > 12,000 \end{cases}$$

where ω_{ij} is the penalty for driving above 12,000 miles, which is equal to zero if the vehicle is bought, but positive if the vehicle is leased. For individuals who drive less than 12,000 miles, operating costs will not affect contract choice as they are identical across lease and buy contracts. For the same reason, the cost of the first 12,000 miles will not affect contract choice for individuals who drive more than 12,000 miles. In the nested logit specification, these common components drop out of the indirect utility and Equation 11 can be written as:

(12) $$\hat{V}_{ij}^{h} = \begin{cases} \alpha_{0i}^{h} + \alpha^{h'}[X_i^h] + \mu_j^h + y^h - \rho^h NP_{ij} - M_{ij} + \kappa^h & \text{if } \theta_0^h \leq 12,000 \\ \alpha_{0i}^{h} + \alpha^{h'}[X_i^h] + \mu_j^h + y^h - \rho^h NP_{ij} - M_{ij} - (\theta_0^h - 12000)\pi_{ij} + \kappa^h & \text{if } \theta_0^h > 12,000 \end{cases}.$$

Based on this, the marginal rate of substitution between maintenance costs and operating costs is

(13) $$\frac{dM_{ij}}{d\pi_{ij}} = -(\theta_0^h - 12000)$$

Similarly, for the general formulation given by Equation 8, the marginal rate of substitution between maintenance cost and operating costs can be derived as:

(14) $$\frac{dM_{ij}}{d\pi_{ij}} = -\frac{\beta_3^h}{\beta_2^h}$$

Comparing Equations 13 and 14, we get

(15) $$-(\theta_0^h - 12000) = -\frac{\beta_3^h}{\beta_2^h},$$

which implies that

(16) $$\theta_0^h = \frac{\beta_3^h}{\beta_2^h} + 12000.$$

The standard deviation of the number of miles is obtained from the ratio of the standard deviation of the operating cost parameter and the maintenance cost parameter, β_2^h.

D. PROFITABILITY ANALYSIS

Let P^R be the retail price of the vehicle. The average wholesale price of the vehicle is obtained by subtracting the dealer margin (PCM^R) from the retail price:

$$P^W = P^R - PCM^R \tag{17}$$

The manufacturer margin is assumed to be 25% of the dealer cost and is therefore given by:

$$PCM^M = .25 * P^W \tag{18}$$

Let q_0^L and q_0^B be the volume of leases and purchases before the promotion is offered. Then, baseline profits are:

$$\pi_0 = PCM_L^M * q_0^L + PCM_B^M * q_0^B \tag{19}$$

where PCM_L^M and PCM_B^M are the margins on leases and purchases respectively.

Let q_{CR}^L and q_{CR}^B be the volume of leases and purchases from a \$x cash rebate on all purchases. Then, total profits from the promotion are given by the expression:

$$\pi_{CR} = PCM_L^M * q_{CR}^L + (PCM_B^M - x) * q_{CR}^B \tag{20}$$

Similarly, if q_{LC}^L and q_{LC}^B be the volume of leases and purchases from a \$x lease cash offered on all leases, profits from the lease cash promotion are given by:

$$\pi_{LC} = (PCM_L^M - x) * q_{LC}^L + PCM_B^M * q_{LC}^B \tag{21}$$

E. CALCULATION OF THE THRESHOLD RETENTION RATE

Let PCM be the price cost margin *before* the promotion is offered. Let q^C and q^R be the total volume from a \$x cash rebate and a \$x residual value enhancement respectively, where $q^C < q^R$. Let λ^* be the fraction of consumers who keep the leased vehicle at the end of the contract, such that the profits from the two promotions are equal:

(22) $$(PCM - x)q^C = [(1-\lambda^*)(PCM - x') + \lambda^* PCM]q^R,$$

where x′ is the discounted residual value promotion, calculated according to Equation 5. Then, the threshold retention rate, λ^* is given by the expression:

(23) $$\lambda^* = \frac{(PCM - x)q^C - (PCM - x')q^R}{x'q^R}.$$

Let λ be the fraction of consumers who actually keep the leased vehicle. Then, if $\lambda^* < \lambda$, then the profit from the residual value enhancement is greater than those from the cash rebate.