**WEB APPENDIX**

# A Dynamic Choice Map Approach to Modeling Attribute-Level Varied Behavior Among Stockkeeping Units

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## Likelihood Function

The log likelihood function is as follows:

$$LL = \sum_{i=1}^{I} \ln \left( \sum_{s=1}^{S} \frac{\exp(\gamma_s)}{\sum_{s'=1}^{S} \exp(\gamma_{s'})} \prod_{t=1}^{T_i} \prod_{j=1}^{J} \left(P_{ijt|s}\right)^{d_{ijt}} \right), \quad (1)$$

where $d_{ijt}$ is a choice indicator, and $\gamma_s$ is the probability to belonging to segment s ( $0 < \gamma_s < 1$ for $\forall s = 1,...,S$ and $\sum_s \gamma_s = 1$ ).

## Issues in Identification

Internal market structure models that use choice data are "under-identified" - these models have more than one solution and parameters need to be constrained to obtain estimates. First, multidimensional latent class models with larger dimensions than the number of segments are not identified (DeSarbo, Manrai, and Manrai 1994; Wedel and DeSarbo 1996). Therefore, the number of latent segments needs to be greater than or equal to the number of dimensions. Second, these models suffer from translational, scale, rotational and reflection indeterminacy (e.g., Erdem 1996; Wedel and DeSarbo 1996). Consistent with prior literature (Andrews and Manrai 1999; Chintagunta 1994), the location for one of the attribute levels for each attribute is constrained at the origin to eliminate translational indeterminacy. The scale, rotation, and reflection indeterminacies come from the fact that the resulting solution can be unchanged via affine transformations (Wedel and Kamakura 2000; Young 1987). This problem is solved by representing the preferences for the first F segments as the location of the K attribute levels in F-dimensional common space, and fixing the location of vectors of these segments at corresponding F dimensions. The remaining (S - F) segment preferences are projected onto these segments' vectors using the attribute-level parameters of the first F segments.

As an example, for a 2 dimensional map model, the preferences for the first two segments form the location for the attribute levels in the 2 dimensional space, with the preference weights for the first two segments being (1,0) and (0,1) respectively. Preference for the third segment would be a linear combination of the preferences for the first two segments, with the weights represented by $(w_1,w_2)$ that need to be estimated from the data. As the weights for the first two segments are constrained to be (1,0) and (0,1) respectively, scale, rotational, and reflection invariance are removed. The parameters of state dependence, $\kappa_{sf}$, are constrained in a similar fashion to remove scale, rotational, and reflection indeterminacy.

**Fit Statistics and Estimates**

Table A1 shows the fit statistics (LL, BIC, CAIC, and ICOMP) for differing numbers of dimensions and segments. All results point to a 2 dimension, 3 segment structure. Table A2 reports the coefficient estimates for each attribute level, variety seeking location, and marketing mix variable for each dimension.

**Table A1**

**Fit Statistics**

| Number of Dimension | Number of Segment | LogL | P | BIC | CAIC | ICOMP |
|---|---|---|---|---|---|---|
| 1 | 1 | -4603 | 25 | 9393 | 9418 | 9207 |
| 1 | 2 | -4571 | 32 | 9380 | 9412 | 9142 |
| 1 | 3 | -4532 | 39 | 9356 | 9395 | 9124 |
| 1 | 4 | -4527 | 46 | 9398 | 9444 | 9130 |
| 2 | 2 | -4509 | 51 | 9399 | 9450 | 9018 |
| **2** | **3** | **-4353** | **64** | **9184** | **9248** | **8706** |
| 2 | 4 | -4351 | 77 | 9278 | 9355 | 8714 |
| 3 | 3 | -4349 | 77 | 9273 | 9350 | 8740 |
| 3 | 4 | -4347 | 96 | 9411 | 9507 | 8744 |

**Table A2**

**Results for 2-Dimension 3-Segment Model**

| | | Dimension 1 | | Dimension 2 | |
|---|---|---|---|---|---|
| Description | Attribute Levels | Coefficient | Std. Error | Coefficient | Std. Error |
| Brand | All Other Brands | 0.292 | 0.123 | -0.796 | 0.173 |
| | Hidden Valley | 0.285 | 0.122 | 0.813 | 0.138 |
| | Kens | 0.266 | 0.114 | -1.007 | 0.208 |
| | Kraft | -0.254 | 0.122 | -0.934 | 0.190 |
| | Wishbone | 0.223 | 0.093 | -1.003 | 0.205 |
| Size | Large | -0.234 | 0.146 | -0.679 | 0.293 |
| | Medium | 1.967 | 0.941 | -5.099 | 1.419 |
| | Small | 0.545 | 0.397 | -2.315 | 0.652 |
| Flavors | All other flavor | 0.272 | 0.087 | 1.312 | 0.931 |
| | Caesar | 0.249 | 0.092 | 0.996 | 0.941 |
| | French/Catalina | -0.377 | 0.172 | 2.557 | 0.958 |
| | Italian Oil/Vinegar | -0.325 | 0.143 | 2.693 | 0.950 |
| | Ranch | -0.336 | 0.146 | 2.517 | 0.936 |
| | Thousand Island | -0.117 | 0.163 | 2.992 | 1.089 |
| Fat | Fat free | 1.465 | 0.482 | -0.896 | 0.591 |
| | Full calorie | -0.265 | 0.144 | 4.305 | 1.119 |
| | gram 2 | 1.800 | 0.598 | -0.207 | 1.669 |
| Weight* | brand weight | -0.413 | 0.459 | -0.266 | 0.340 |
| | size weight | -13.534 | 19.558 | -6.429 | 9.454 |
| | flavor weight | 0.400 | 1.674 | 1.286 | 0.466 |
| | fat weight | -1.137 | 0.639 | 0.426 | 0.164 |
| Variety seeking Location | Brand | -11.657 | 10.464 | -2.597 | 1.090 |
| | Size | 0.628 | 0.443 | -0.084 | 0.020 |
| | Flavors | -3.315 | 2.767 | 0.053 | 0.029 |
| | Fat | 0.576 | 0.224 | 0.118 | 0.030 |
| Weight* | Variety seeking | -0.454 | 0.295 | 0.673 | 0.189 |
| Marketing Mix | Price | -0.286 | 0.192 | 1.347 | 0.363 |
| | PPI | -0.863 | 0.220 | -1.474 | 0.429 |
| | Feature | 1.226 | 0.172 | -0.235 | 0.392 |
| | Display | 0.687 | 0.229 | 0.327 | 0.557 |
| Weight* | Marketing Mix | 1.495 | 0.279 | -0.657 | 0.333 |
| Segment Weight | Segment 2 | -0.970 | 0.221 | | |
| | Segment 3 | -0.943 | 0.223 | | |

* Weights for the first two segments for the attributes, marketing mix and variety seeking parameters are constrained to be equal to (1,0) for Segment 1 and (0,1) for Segment 2.

## References

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