Brand-Level Effects of Stockkeeping Unit Reductions

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Web Appendix: The Joint Model of Purchase Incidence, Brand Choice, and Purchase Quantity

We extend the model developed by Zhang and Krishnamurthi (2004) to allow a more flexible distribution of the error terms.

Define: I_{it} = 1 if household *i* makes a category purchase in week *t*, 0 otherwise; B_{ikt} = 1 if household *i* purchases brand *k* in week *t*, 0 otherwise; and Q_{ikt} = household *i*'s purchase quantity of brand *k* in week *t*.

In generic terms, the brand utility functions are given by: $U_{ikt} = V_{ikt} + \varepsilon_{ikt}$, where k = 1, …, K denotes brands; the category purchase threshold function is: $U_{i0t} = V_{i0t} + \varepsilon_{i0t}$; and the latent purchase quantity function is: $Q^*_{ikt} = Z_{ikt} + \xi_{ikt}$, where ξ_{ikt} follows a logistic distribution with mean 0, scale parameter δ_ξ, and cumulative distribution function (CDF):

$$\text{(A1)} \qquad F(\xi_{ikt}) = \frac{1}{1+\exp(-\delta_\xi \xi_{ikt})} .$$

Household *i* makes a category purchase at *t* if and only if $\max(U_{ikt}, k = 1,..., K) > U_{i0t}$, chooses brand *k* at *t* if and only if $U_{ikt} > \max(U_{ijt}, j \neq k \text{ and } j = 0,1,..., K)$, and the observed household purchase quantity is $Q_{ikt} = Q^*_{ikt}$ if I_{it} = 1 and B_{ikt} = 1, and Q_{ikt} = 0 otherwise.

Similar to Bell, Chiang, and Padmanabhan (1999), we use a nested logit model derived from a Generalized Extreme Value distribution for purchase incidence and brand choice. Specifically, we assume that the error terms ε_{ikt}, k = 0, 1, …, K, follow a joint Generalized Extreme Value (GEV) distribution with the joint CDF:

$$\text{(A2)} \qquad F(\varepsilon_0, \varepsilon_1, ..., \varepsilon_K) = \exp\{-G(e^{-\underline{\varepsilon}})\} = \exp\{-[\textstyle\sum_{j=1}^{K} e^{-\varepsilon_j}]^{1-\phi} - e^{-\varepsilon_0}\} , \ 0 < \phi < 1 .$$

It can be shown that (see McFadden 1978, and Ben-Akiva and Lerman 1985):

(A3) $$\Pr\{I_{it}=1\}=\frac{\exp\{(1-\phi)\ln[\sum_{j=1}^{K}e^{V_{ijt}}]\}}{e^{V_{i0t}}+\exp\{(1-\phi)\ln[\sum_{j=1}^{K}e^{V_{ijt}}]\}}=\frac{[\sum_{j=1}^{K}e^{V_{ijt}}]^{1-\phi}}{e^{V_{i0t}}+[\sum_{j=1}^{K}e^{V_{ijt}}]^{1-\phi}}$$, and

(A4) $$\Pr\{\,I_{it}=1,B_{ikt}=1\}=\frac{e^{V_{ikt}}[\sum_{j=1}^{K}e^{V_{ijt}}]^{-\phi}}{e^{V_{i0t}}+[\sum_{j=1}^{K}e^{V_{ijt}}]^{1-\phi}}.$$

This is a nested logit model with the "inclusive value" equal to $\ln\left(\sum_{j=1}^{K}e^{V_{ijt}}\right)$ and its coefficient being $(1-\phi)$ (McFadden 1978). The parameter ϕ measures the degree of similarity among the choice alternatives.

The joint probability in equation (A7) can also be derived from the following.

(A5) $$\begin{aligned}\Pr\{I_{it}=1,B_{ikt}=1\}&=\Pr\{U_{ikt}>\max(U_{ijt},\, j\neq k \text{ and } j=0,1,\ldots,K)\}\\&=\Pr\{V_{ikt}+\varepsilon_{ikt}>\max(V_{ijt}+\varepsilon_{ijt},\, j\neq k \text{ and } j=0,1,\ldots,K)\}\\&=\Pr\{\max(V_{ijt}+\varepsilon_{ijt},\, j\neq k \text{ and } j=0,1,\ldots,K)-\varepsilon_{ikt}<V_{ikt})\}\end{aligned}$$

Let $\varepsilon_{ikt}^{*}=\max(V_{ijt}+\varepsilon_{ijt},\, j\neq k \text{ and } j=0,1,\ldots,K)-\varepsilon_{ikt}$. It can be shown that the CDF of ε_{ikt}^{*} is (see Bell, Chiang, and Padmanabhan 1999, page 524):

(A6) $$F(\varepsilon_{ikt}^{*})=\frac{e^{\varepsilon_{ikt}^{*}}[e^{\varepsilon_{ikt}^{*}}+\sum_{j\neq k,\, j=1,\ldots,K}e^{V_{ijt}}]^{-\phi}}{e^{V_{i0t}}+[e^{\varepsilon_{ikt}^{*}}+\sum_{j\neq k,\, j=1,\ldots,K}e^{V_{ijt}}]^{1-\phi}}.$$

It is straightforward to see that $\Pr\{I_{it}=1,B_{ikt}=1\}=\Pr\{\varepsilon_{ikt}^{*}<V_{ikt}\}=F(\varepsilon_{ikt}^{*}=V_{ikt})$.

We accommodate the independence in the purchase incidence, brand choice, and purchase quantity decisions by allowing ε_{ikt}^{*} and ξ_{ikt} to be jointly distributed. We adopt an approach introduced by Morgenstern (1956) and late extended by Farlie (1960) and Plackett (1965) to constructing a joint bivariate distribution function from known marginal distributions. The joint CDF is given by:

(A7) $$F(\varepsilon_{ikt}^{*},\xi_{ikt})=F(\varepsilon_{ikt}^{*})F(\xi_{ikt})[1+\theta(1-F(\varepsilon_{ikt}^{*}))(1-F(\xi_{ikt}))],\ -1\leq\theta\leq1,$$

where θ is to be estimated from the data. $\theta = 0$ means that the two random terms are independent. By the definition of ε^*_{ikt}, $\theta > 0$ indicates that unobserved factors in the quantity and choice functions are *negatively* correlated, and $\theta < 0$ indicates that unobserved factors in the two components are *positively* correlated (see Zhang and Krishnamurthi 2004).

We now derive the joint probability of $\Pr(I_{it} = 1, B_{ikt} = 1, Q_{ikt} = q_{ikt})$. For notation simplicity, let $A = F(\varepsilon^*_{ikt} = V_{ikt}) = \Pr\{I_{it} = 1, B_{ikt} = 1\}$.

(A8)
$$\begin{aligned}
&\Pr\{I_{it} = 1, B_{ikt} = 1, Q_{ikt} = q_{ikt}\} \\
&= \int_{-\infty}^{V_{ikt}} f(\varepsilon^*_{ikt}, \xi_{ikt} = q_{ikt} - Z_{ikt}) d\varepsilon^*_{ikt} \\
&= \frac{\partial F(\varepsilon^*_{ikt}, \xi_{ikt})}{\partial \xi_{ikt}}\Bigg|_{\substack{\varepsilon^*_{ikt} = V_{ikt} \\ \xi_{ikt} = q_{ikt} - Z_{ikt}}} \\
&= \frac{\partial}{\partial \xi_{ikt}}\left\{F(\varepsilon^*_{ikt})\frac{1}{1+e^{-\delta_\xi \xi_{ikt}}}[1+\theta(1-F(\varepsilon^*_{ikt}))(1-\frac{1}{1+e^{-\delta_\xi \xi_{ikt}}})]\right\}\Bigg|_{\substack{\varepsilon^*_{ikt} = V_{ikt} \\ \xi_{ikt} = q_{ikt} - Z_{ikt}}}, \\
&= \frac{\partial}{\partial \xi_{ikt}}\left\{A\frac{1}{1+e^{-\delta_\xi \xi_{ikt}}} + \theta \cdot A(1-A)\frac{e^{-\delta_\xi \xi_{ikt}}}{(1+e^{-\delta_\xi \xi_{ikt}})^2}\right\}\Bigg|_{\xi_{ikt} = q_{ikt} - Z_{ikt}} \\
&= A\frac{\delta_\xi e^{\delta_\xi (Z_{ikt} - q_{ikt})}}{[1+e^{\delta_\xi (Z_{ikt} - q_{ikt})}]^2}[1+\theta(1-A)\frac{-1+e^{\delta_\xi (Z_{ikt} - q_{ikt})}}{1+e^{\delta_\xi (Z_{ikt} - q_{ikt})}}]
\end{aligned}$$

where $A = \dfrac{e^{V_{ikt}}[\sum_{j=1}^{K} e^{V_{ijt}}]^{-\phi}}{e^{V_{i0t}} + [\sum_{j=1}^{K} e^{V_{ijt}}]^{1-\phi}}$.

It can be shown that the conditional expectation of purchase quantity is:

(A9) $$E(Q_{ikt} \mid I_{it} = 1, B_{ikt} = 1) = \frac{1}{\delta_\xi}\left[\log(1+e^{\delta_\xi Z_{ikt}}) - \theta \cdot (1-A)\frac{e^{\delta_\xi Z_{ikt}}}{1+e^{\delta_\xi Z_{ikt}}}\right],$$

where A is the joint probability of purchase incidence and brand choice specified previously (see Zhang and Krishnamurthi 2004).

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Additional Table 1
Average Prices, Price Cuts, and Market Share for Brands Studied

	Regular Price (cents/oz.)		*Price Cut (cents/oz.)*		*Shelf Price (cents/oz.)*		*Market Share*	
	Before SR	After SR	Before SR	After SR	Before SR	After SR	Before SR	After SR
Liquid Detergent								
Wisk	7.06	8.01	.94	1.26	6.12	6.75	10.9%	12.9%
All	4.50	5.20	.38	.55	4.11	4.65	6.3%	10.9%
Tide	6.98	7.84	.57	.59	6.42	7.25	51.9%	48.7%
Cheer	6.64	7.07	.16	0	6.48	7.07	8.4%	6.5%
Arm & Hammer	4.93	5.01	.45	.60	4.48	4.41	5.7%	4.9%
Era	5.90	6.01	.01	.14	5.89	5.87	4.6%	3.9%
Dreft	9.73	10.08	0	0	9.73	10.08	6.6%	4.4%
Surf	6.00	6.65	.09	.63	5.91	6.02	3.3%	3.1%
Private label	4.43	3.91	.10	.11	4.32	3.79	2.5%	4.7%
Overall	6.74	7.19	.47	.57	6.27	6.62	100%	100%
Margarine								
Brummel & Brown	14.75	15.66	.33	.71	14.43	14.95	6.2%	6.4%
Fleischmann’s	13.56	17.82	.02	1.77	13.55	16.05	9.5%	9.7%
I Can’t Believe…	14.83	16.12	.76	.79	14.07	15.33	26.5%	27.1%
Imperial	10.26	11.23	.11	.18	10.15	11.04	13.0%	9.5%
Land O’Lakes	13.56	16.33	.24	.40	13.32	15.93	8.3%	10.5%
Parkay	11.38	13.08	.48	.19	10.90	12.89	7.2%	7.4%
Promise	13.81	17.49	.23	.71	13.58	16.77	8.9%	12.6%
Shedds Country	6.98	7.96	.09	.52	6.88	7.44	15.7%	13.0%
Private label	5.81	5.60	.90	.30	4.91	5.3	4.7%	3.9%
Overall	12.01	14.31	.37	.67	11.64	13.63	100%	100%
Spaghetti Sauce								
Barilla	9.78	10.85	.19	.45	9.58	10.40	5.4%	8.1%
Classico	10.97	11.98	.22	.25	10.75	11.74	15.3%	13.2%
Five Brothers	11.19	11.97	.37	.35	10.81	11.62	4.0%	4.8%
Healthy Choice	8.50	9.40	.41	.49	8.09	8.91	1.9%	3.8%
Hunt’s	4.59	5.00	.30	.29	4.29	4.7	1.9%	1.8%
Newman’s Own	8.95	9.47	.14	.22	8.82	9.24	5.9%	5.5%
Prego	7.87	8.18	.17	.48	7.70	7.70	34.9%	39.8%
Ragu	7.38	8.60	.43	.32	6.95	8.28	28.0%	20.7%
Private label	7.61	6.81	.86	.81	6.75	5.99	2.7%	2.3%
Overall	8.45	9.20	.28	.40	8.17	8.80	100%	100%

Additional Table 2
Pearson Correlation Coefficients of Variables in the Second-Stage Analysis

Variables in Models 1 and 2

	ΔCQ	γ^*	*FRQ*	*BOUT*	*SOUT*	*EHSHR*
ΔPRI	.3804	.7970	.1439	-.0031	-.0119	-.0496
ΔCQ		.3461	.0442	-.0504	-.0041	-.0469
γ^*			.1505	.0108	-.0744	-.1139
FRQ				-.0334	-.0835	-.0397
BOUT					-.0129	.0647
SOUT						.7235

Variables in Model 3

	MS	*PRICE*	*LPROM*	*DSKU*	*DSIZE*	*ΔSKUSHR*	*EBSHR*
ΔPRB	.1326	.1472	.0386	-.0656	-.1688	.1115	.0163
MS		.0901	.1880	.4237	.2287	.2009	-.1892
PRICE			-.4601	-.0787	-.1931	.0905	-.0942
LPROM				.5401	.1338	-.2097	.3348
DSKU					.5856	-.5855	.4750
DSIZE						-.4983	.0930
ΔSKUSHR							-.5406

ΔPRI = standardized change in category purchase incidence probability;
ΔCQ = standardized change in quantity given a purchase occasion;
γ^* = standardized household state dependence measure;
FRQ = standardized category purchase frequency;
BOUT = favorite brand being eliminated;
SOUT = favorite SKU being eliminated;
EHSHR = eliminated SKUs' share of household purchase quantity;
ΔPRB = change in conditional brand choice probability;
MS = market share before the SR;
PRICE = standardized price level before the SR;
LPROM = logarithm of promotion frequency before the SR;
DSKU = number of SKUs eliminated;
DSIZE = number of sizes eliminated;
ΔSKUSHR = change in the share of SKUs;
EBSHR = eliminated SKUs' share of brand sales;