Consumer Value-Maximizing Sweepstakes and Contests

Ajay Kalra, Mengze Shi

Web Appendix

Web Appendix A: Sweepstakes for High-brand valuation Segment

Optimality condition

(Problem 1) $$\underset{s}{\text{Max}}\, V_H(S) = \omega(\frac{m_1}{N})g(r_1) + \sum_{j=2}^{n}\left(\omega(\frac{\sum_{k=1}^{j} m_k}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k}{N})\right)g(r_j) + \left(1-\omega(\frac{\sum_{j=1}^{n} m_j}{N})\right)g(0)$$

$$\text{s.t. } \sum_{j=1}^{n} m_j r_j = R$$

We characterize value-maximizing sweepstakes based on optimality conditions. Let the value-maximizing sweepstakes be $S^* = \{r_1^*, m_1^*; r_2^*, m_2^*; ...r_n^*, m_n^*\}$. Consider a very small amount of prize (∇r) reduced from r_k^* and allocated to r_j^* ($1 \le j, k \le n, j \ne k$), while prizes of all other ranks remains the same. With total budget R fixed, r_k^* should decrease by $m_j \nabla r / m_k$. Therefore, $dr_k / dr_j = -m_j / m_k$. Such an allocation should not change anticipated value from sweepstakes, that is,

$$\frac{dV_H(S^*)}{dr_j} = \omega_j g'(r_j^*) + \omega_k g'(r_k^*)\frac{dr_k}{dr_j} = m_j\left[\frac{\omega_j}{m_j}g'(r_j^*) - \frac{\omega_k}{m_k}g'(r_k^*)\right] = 0 \qquad \text{(A1)}$$

Equation (A1) leads to optimality condition for value-maximizing sweepstakes of Problem 1:

$$\frac{\omega_j}{m_j}g'(r_j^*) = M_H^* \quad (j = 1,2,..n) \qquad \text{(A2)}$$

Proof that there is only one winner at each rank when consumers are risk averse in gain

Suppose there are two winners at j^{th} rank for prize r_j. Consumer's anticipated value from this rank of prize is equal to

$$\left(\omega\left(\left(\sum_{k=1}^{j-1} m_k + 2\right)\Big/ N\right) - \omega\left(\left(\sum_{k=1}^{j-1} m_k\right)\Big/ N\right)\right)g(r_j) =$$

$$\left(\omega(\frac{\sum_{k=1}^{j-1} m_k + 1}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k}{N})\right)g(r_j) + \left(\omega(\frac{\sum_{k=1}^{j-1} m_k + 2}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k + 1}{N})\right)g(r_j)$$

For a positive and sufficiently small σ, we can reallocate prize ($2r_j$) into (r_j -σ) and (r_j +σ), keeping other prizes and their associated decision weights unchanged. With such a change in prize structure, the anticipated value from these two prizes become

$$\left(\omega(\frac{\sum_{k=1}^{j-1} m_k + 1}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k}{N})\right) g(r_j + \sigma) + \left(\omega(\frac{\sum_{k=1}^{j-1} m_k + 2}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k + 1}{N})\right) g(r_j - \sigma)$$

Then the change in anticipated value of sweepstakes resulting from prize reallocation is

$$\left(\omega(\frac{\sum_{k=1}^{j-1} m_k + 1}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k}{N})\right) \left(g(r_j + \sigma) - g(r_j)\right) - \left(\omega(\frac{\sum_{k=1}^{j-1} m_k + 2}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k + 1}{N})\right) \times$$
$$\left(g(r_j) - g(r_j - \sigma)\right) \qquad \text{(A3)}$$

In (A3), according to equation (6), $\omega\left(\left(\sum_{k=1}^{j-1} m_k + 1\right) \Big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \Big/ N\right)$ is larger than $\omega\left(\left(\sum_{k=1}^{j-1} m_k + 2\right) \Big/ N\right) - \omega\left(\left(\sum_{k=1}^{j-1} m_k + 1\right) \Big/ N\right)$. Moreover, *the difference is strictly positive and independent of σ.* On the other hand, when σ becomes smaller, the difference between $\left(g(r_j) - g(r_j - \sigma)\right)$ and $\left(g(r_j + \sigma) - g(r_j)\right)$ decreases and eventually approaches to zero. Therefore, there exists a positive and sufficiently small σ^* so that (A3) is positive for any $0<\sigma<\sigma^*$. In other words, the firm can increase the anticipated value of sweepstakes promotion by reallocating ($2r_j$) into (r_j-σ) and (r_j+σ) as long as σ is small enough. Thus, it is not value-maximizing to have a rank with more than one winner. Instead, only a single winner should be awarded for every level of prize.

Web Appendix B: Sweepstakes for the Low-brand valuation Segment

(Problem 2)

$$\begin{aligned} \underset{s}{Max}\, V_L(S) &= \omega\left(\frac{m_1}{N}\right) g(r_1 - \tau) + \sum_{j=2}^{J} \left(\omega\left(\sum_{k=1}^{j} m_k \Big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \Big/ N\right)\right) g(r_j - \tau) \\ &- \sum_{j=J+1}^{n} \left(\omega\left(\sum_{k=1}^{j} m_k \Big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \Big/ N\right)\right) l(\tau - r_j) - \left(1 - \omega\left(\sum_{j=1}^{n} m_j \Big/ N\right)\right) l(\tau) \end{aligned}$$

s.t. $\sum_{j=1}^{n} m_j r_j = R$

Proof that the lowest prize should be at least as large as opportunity cost τ.

Suppose a sweepstake S includes a prize smaller than opportunity cost τ. Without loss of generality, we let $S = \{r_1, m_1; r_2, m_2; ...; r_n, m_n\}$ where $r_n m_n = R_n$, $\sum_{j=1}^{n} m_j r_j = R$, and $r_n < \tau$. Now we show that under the same budget (R), we can enhance the valuation of sweepstakes with an increase in lowest reward r_n. Keeping $\{r_1, m_1; r_2, m_2; ...; r_{n-1}, m_{n-1}\}$ the same, we let r_n increase by a very small amount while keeping R_n (hence total expense R) unchanged. With a constant R_n, since $m_n = R_n \big/ r_n$, an increase in r_n implies a decrease in m_n. An increase in r_n will lead to following changes in the low-brand valuation consumers' anticipated value from sweepstakes participation:

$$\frac{\partial V_L(S)}{\partial r_n} = \frac{\partial \omega_n}{\partial r_n}\left[-l(\tau - r_n)\right] + \omega_n l'(\tau - r_n) + \frac{\partial \omega_n}{\partial r_n}\left[l(\tau)\right]$$

$$= \frac{m_n}{N}\left[\frac{\omega(\sum_{j=1}^{n} m_j \big/ N) - \omega(\sum_{j=1}^{n-1} m_j \big/ N)}{m_n / N} l'(\tau - r_n) - \omega'\left(\sum_{j=1}^{n} m_j \Big/ N\right)\frac{l(\tau) - l(\tau - r_n)}{r_n}\right] > 0 \qquad \text{(A4)}$$

The above inequality always holds because a) $\frac{\omega(\sum_{j=1}^{n} m_j \big/ N) - \omega(\sum_{j=1}^{n-1} m_j \big/ N)}{m_n / N} > \omega'\left(\sum_{j=1}^{n} m_j \Big/ N\right)$ due to the *s*-shaped decision weighting function, and b) $l'(\tau - r_n) > \frac{l(\tau) - l(\tau - r_n)}{r_n}$ due to the concavity of the loss function. Since $\frac{\partial V_L(S)}{\partial r_n} > 0$, sweepstakes valuation can be enhanced with an increase in r_n. Therefore a sweepstakes $S = \{r_1, m_1; r_2, m_2; ...; r_n, m_n\}$ with $r_n < \tau$ is not value-maximizing. The lowest prize in value-maximizing sweepstakes should be at least as large as opportunity cost τ.

Optimality Condition

As in Web Appendix A, we characterize value-maximizing sweepstakes based on optimality conditions. Let the value-maximizing sweepstakes be $S^* = \{r_1^*, m_1^*; r_2^*, m_2^*; ...r_n^*, m_n^*\}$. Since the smallest prize should be as large as τ, we let $r_n^* = \tau$. We now consider a very small amount of prize (∇r) *deducted from* r_k^* and allocated to r_j^* ($1 \le j$, $k \le n$, $j \ne k$), while prizes of all other ranks remain the same. With total budget R fixed, r_k^* should increase by $m_j \nabla r \big/ m_k$. Therefore, $dr_k \big/ dr_j = -m_j \big/ m_k$. Anticipated value from sweepstakes would then change by:

$$\frac{dV_L(S^*)}{dr_j} = \omega_j g'(r_j^* - \tau) + \omega_k g'(r_k^* - \tau)\frac{dr_k}{dr_j} = m_j\left[\frac{\omega_j}{m_j} g'(r_j^* - \tau) - \frac{\omega_k}{m_k} g'(r_k^* - \tau)\right] = 0 \quad (j, k \ne n) \qquad \text{(A5)}$$

$$\frac{dV_L(S^*)}{dr_n} = \omega_n g'(r_n^* - \tau) + \omega_k g'(r_k^* - \tau)\frac{dr_k}{dr_n} = m_n\left(\frac{\omega_n}{m_n} g'(0) - \frac{\omega_k}{m_k} g'(r_k^* - \tau)\right), \ (j=n) \qquad \text{(A6)}$$

$$\frac{dV_L(S^*)}{dr_j} = \omega_j g'(r_j^* - \tau) + \omega_n l'(r_k^* - \tau)\frac{dr_n}{dr_j} = m_j\left(\frac{\omega_j}{m_j} g'(r_j^* - \tau) - \frac{\omega_n}{m_n} l'(0)\right), \ (k=n) \qquad \text{(A7)}$$

Equation (A5) characterizes optimality condition for first (n-1) prizes,

$$\frac{\omega_j}{m_j} g'(r_j^* - \tau) = M_L^* \quad (j = 1,2,..n-1) \qquad \text{(A8)}$$

Similar to (A2), (A8) requires identical anticipated value generating ability M_L^* from the top (n-1) prizes. At the bottom prize that equals to switching cost τ, anticipated value is not differentiable because of loss aversion. The value-maximizing sweepstakes contain a bottom prize equal to τ if

$$\frac{\omega_n}{m_n} l'(0) \geq M_L^* \geq \frac{\omega_n}{m_n} g'(0) \tag{A9}$$

Condition (A9) ensures that reducing the lowest prize will reduce the anticipated value.

When low-brand valuation consumers are risk-averse in gain, (A8) and (A9) indicate that the value-maximizing sweepstakes is to have multiple big prizes in addition to the bottom prize equal to τ. Following exactly the same logic as given in second part of Web Appendix A, we can show that number of winners for each rank of big prizes should be equal to one. Then the optimality condition (A8) becomes:

$$\left(\omega(\frac{j}{N}) - \omega(\frac{j-1}{N})\right) g'(r_j^* - \tau) = M_L^* \ \ (j = 1,2,..n\text{-}1) \tag{A10}$$

Value-maximizing number of lowest-prize winners

We now analyse the value-maximizing number of winners for the lowest prize (τ). Consider a very small increase (∇m) in number of last-prize winners (m_n^*) and a decrease in r_j^* ($1 \leq j \leq n\text{-}1$), while keeping prizes of all other ranks the same. To maintain the same total budget R, r_j^* should decrease by $\tau \nabla m$. Therefore, $\partial r_j / \partial m_n = -\tau$. Such a reallocation should not change the low-brand valuation consumers' anticipated value of sweepstakes:

$$\begin{aligned} \frac{\partial V_L(S^*)}{\partial m_n} &= \omega_j g'(r_j^* - \tau) \frac{\partial r_j}{\partial m_n} + \frac{\partial \omega\left(\frac{n-1+m_n^*}{N}\right)}{\partial m_n} \big(l(\tau) - l(0)\big) \\ &= \tau \left(\frac{l(\tau)}{\tau} \frac{\omega'\left(\frac{n-1+m_n^*}{N}\right)}{N} - \omega_j g'(r_j^* - \tau) \right) = 0 \end{aligned} \tag{A11}$$

Combining condition (A11) with (A10), we have the following optimality condition:

$$\left(\omega\left(\frac{j}{N}\right) - \omega\left(\frac{j-1}{N}\right)\right) g'(r_j^* - \tau) = \frac{l(\tau)}{\tau} \frac{\omega'\left(\frac{n-1+m_n^*}{N}\right)}{N} \quad (j=1,2,\ldots n\text{-}1)$$

which can be rewritten as

$$\omega'\left(\frac{n-1+m_n^*}{N}\right) = \frac{\omega\left(\frac{j}{N}\right) - \omega\left(\frac{j-1}{N}\right)}{1/N} \frac{g'(r_j - \tau)}{l(\tau)/\tau} \qquad (j=1,2,\ldots n\text{-}1) \tag{A12}$$

In the special case of low-brand valuation consumers being risk-neutral in gain, the value-maximizing sweepstakes only offers one big prize; that is, n=2. Then condition (A12) simplifies to:

$$\omega'\left(\frac{1+m_2^*}{N}\right) = \frac{\omega\left(\frac{1}{N}\right)}{1/N} \frac{g'(r_1^* - \tau)}{l(\tau)/\tau} \tag{A13}$$

Web Appendix C

Table A: Parameter Estimates for Pairwise Logit Models (Cookies)*

M& M Sweepstakes							
n=76	Sweepstakes Pair	Sigma (σ)		Alpha (α)		I(LowValue)	
		Coeff	*T*	coeff	*t*	coeff	*t*
MLP& Small vs GP	(20,$25; 250,$2) > (1, $1000)	-4.340	-1.82	**5.553**	**3.03**	**1.432**	**2.33**
MLP& Small vs MLP	(20,$25;250,$2)>(10,$50; 20,$25)	**-4.823**	**-1.99**	**5.834**	**3.02**	**2.645**	**3.95**
MLP&Small vs GP& Small	(20,$25;250,$2)>(1,$500;250,$2)	**-9.124**	**-3.54**	**3.447**	**2.15**	0.058	0.1
MLP vs GP	(10, $50; 20,$25) > (1, $1000)	-0.541	-0.27	1.693	1.14	-0.509	-0.9
GP&Small vs GP	(1, $500; 250,$2) > (1, $1000)	-0.234	-0.12	**3.560**	**2.41**	0.855	1.54
GP&Small vs MLP	(1,$500;250,$2)>(10,$50;20,$25)	3.881	1.75	0.492	0.32	**2.185**	**3.72**
Fig Newtons Sweepstakes							
MLP&Small vs GP	(20,$25; 250,$2) > (1, $1000)	**-7.114**	**-3**	**4.275**	**2.63**	**1.233**	**2.28**
MLP&Small vs MLP	(20,$25; 250,$2) > (10, $50; 20, $25)	**-4.895**	**-2.28**	2.435	1.69	**1.512**	**2.84**
MLP&Small vs GP&Small	(20,$25; 250,$2)>(1,$500;250,$2)	**-10.583**	**-3.8**	**5.123**	**2.94**	0.241	0.44
MLP vs GP	(10, $50; 20,$25) > (1, $1000)	**-6.630**	**-2.79**	**3.227**	**2.03**	-0.337	-0.62
GP&Small vs GP	(1, $500; 250,$2) > (1, $1000)	**-5.948**	**-2.58**	**3.176**	**2**	**1.714**	**3.1**
GP&Small vs MLP	(1,$500;250,$2)>(10,$50;20,$25)	-1.403	-0.73	-0.197	-0.15	**1.596**	**3.06**

*Numbers in bold indicate significant estimates.

Table B: Parameter Estimates for Pairwise Logit Models (Mints)*

n=76	Sweepstakes Pair	Sigma (σ)		Alpha (α)		I(LowValue)	
		Coeff	*t*	coeff	*t*	coeff	*t*
Lifesaver Sweepstakes							
MLP&Small vs GP	(20,$15; 200,$1) > (1, $500)	**-14.704**	**-3.9**	**7.037**	**3.29**	-0.263	-0.41
MLP&Small vs MLP	(20,$15; 200,$1) > (1,$200; 1, $150; 1, $100; 1, $50)	**-12.763**	**-3.78**	**5.783**	**2.98**	0.326	0.55
MLP&Small vs GP&Small	(20,$15;200,$1)>(1,$300;200,$1)	**-11.094**	**-3.64**	**4.690**	**2.67**	0.266	0.46
MLP vs GP	(1, $200; 1, $150; 1, $100;1, $50) > (1, $500)	-0.656	-0.34	1.394	1.05	**1.142**	**2.08**
GP&Small vs GP	(1, $300; 200,$1) > (1, $500)	**-7.126**	**-2.84**	**4.281**	**2.66**	0.900	1.62
GP&Small vs MW	(1,$300; 200,$1) > (1,$200; 1, $150; 1, $100; 1, $50)	**-5.428**	**-2.4**	2.350	1.64	0.762	1.42
Altoids Sweepstakes							
MLP&Small vs GP	(20,$15; 200,$1) > (1, $500)	-1.341	-0.63	2.543	1.83	0.747	1.26
MLP&Small vs MLP	(20,$15; 200,$1) > (1,$200; 1, $150; 1, $100; 1, $50)	-2.946	-1.29	4.008	2.58	1.501	2.39
MLP&Small vs GP&Small	(20,$15; 200,$1) > (1,$300; 200,$1)	**-7.236**	**-2.96**	**3.354**	**2.24**	-0.175	-0.3
MLP vs GP	(1, $200; 1, $150; 1, $100;1, $50) > (1, $500)	-4.437	-1.92	-0.502	-0.35	**-1.592**	**-2.71**
GP&Small vs GP	(1, $300; 200,$1) > (1, $500)	-1.419	-0.66	**3.797**	**2.55**	0.978	1.62
GP&Small vs MW	(1,$300; 200,$1) > (1,$200; 1, $150; 1, $100; 1, $50)	-1.495	-0.61	**4.908**	**2.77**	**2.233**	**3.22**

*Numbers in bold indicate significant estimates.