Early Marketing Matters:
A Time-Varying Parameter Approach to Persistence Modeling

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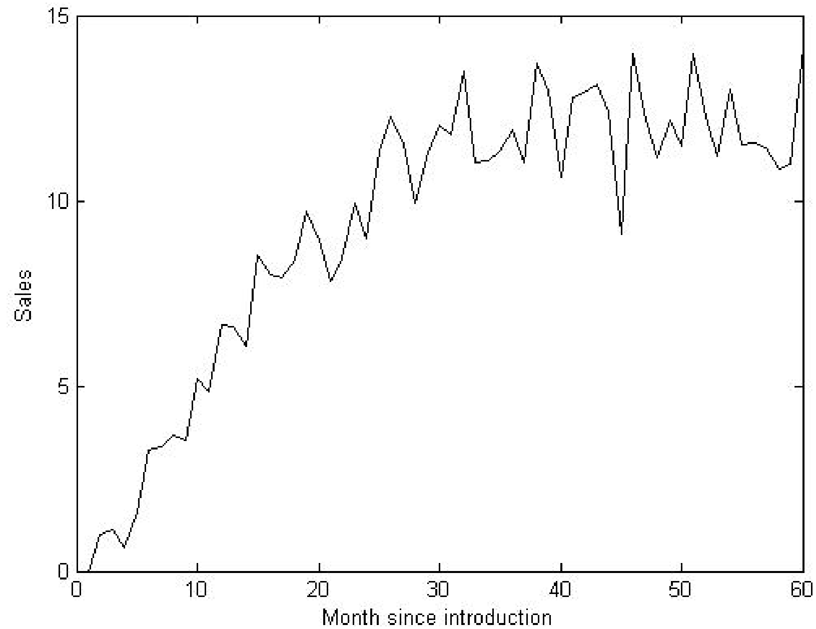
WEB APPENDIX A

We show that the outcome of a unit root test can depend on the timeframe chosen. In Figure A1 we depict a hypothetical sales curve generated from normally distributed random numbers. The data generation process for the first 30 observations is a local level model, i.e. this part of the series is nonstationary. In the subsequent 30 periods sales are stationary. Unit root tests confirm this structure: for the first 30 observations, ADF tests, which assume a constant only or a constant and a trend, cannot reject the null hypothesis of a unit root, but the same tests applied to the last 30 observations strongly reject the null hypothesis. When applied to the entire sample, the tests provide mixed results. Specifically, when assuming a constant only, the ADF test provides strong evidence against the presence of a unit root ($p < .01$), but when it assumes a constant and a trend, it cannot reject the null hypothesis ($p = .71$). Thus, the conventional methodology suggests the presence of persistent effects for the first 30 observations, but when it analyzes the entire sample and assumes a constant only, it indicates that persistent effects are absent. This example stresses the restrictive character of conventional methodology: Persistent effects must either exist for all time periods or be totally absent; they cannot be present for a few time periods only. In addition, allowing for exogenous (Perron 1989) or endogenous (Lee and Strazicich 2001) structural breaks cannot solve this problem, because doing so accommodates for changes in the level or slope of the trend function but does not allow for a series that consists of both nonstationary and stationary parts.

FIGURE A1

SALES SERIES CONSISTING OF A NONSTATIONARY (1-30) AND A STATIONARY PART (31-60)



WEB APPENDIX B

Filtering and smoothing routines require writing the state space model in matrix form. A typical state space model takes the following form (e.g., Durbin and Koopman 2001, ch. 3):

Measurement equation:

(A2) $$y_t = Z_t \alpha_t + G_t \varepsilon_t .$$

Transition equation:

(A3) $$\alpha_t = T_t \alpha_{t-1} + H_t \eta_t .$$

In these equations, $\varepsilon_t \sim N(0, I)$, and $\eta_t \sim N(0, I)$ for $t = 1, \ldots n$.

In Equations A4 and A5, we represent the basic model from Equations 1–6 in matrix form:

Measurement equation:

(A4) $$y_t = \begin{bmatrix} 1 & 0 & 0 & x_t & 0 \end{bmatrix} \begin{bmatrix} \beta_{0t} \\ \beta_{2t} \\ \pi_{2t} \\ \beta_{1t} \\ \pi_{1t} \end{bmatrix} + \varepsilon_t .$$

Transition equation:

(A5) $$\begin{bmatrix} \beta_{0t} \\ \beta_{2t} \\ \pi_{2t} \\ \beta_{1t} \\ \pi_{1t} \end{bmatrix} = \begin{bmatrix} 1 & x_t & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_{0t-1} \\ \beta_{2t-1} \\ \pi_{2t-1} \\ \beta_{1t-1} \\ \pi_{1t-1} \end{bmatrix} + \begin{bmatrix} \sigma_{\eta_0} & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\eta_1} & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\eta_2} & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\eta_3} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\eta_4} \end{bmatrix} \begin{bmatrix} \eta_{0t} \\ \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \\ \eta_{4t} \end{bmatrix} .$$

Again, $\varepsilon_t \sim N(0, I)$, and $\eta_t \sim N(0, I)$ for $t = 1, \ldots n$.

REFERENCES

Durbin, James and Siem J. Koopman (2001), *Time Series Analysis by State Space Methods*. New York: Oxford University Press.

Lee, Junsoo and Mark Strazicich (2001), "Testing the Null of Stationarity in the Presence of a Structural Break," *Applied Economics Letters*, 8 (6), 377–82.

Perron, Pierre (1989), "The Great Crash, the Oil Price Shock and the Unit-Root Hypothesis," *Econometrica*, 57 (6), 1361–1401.