Web Appendix

Specifying Price Judgments with Range–Frequency Theory in Models of Brand Choice

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Section A – Constructing Memory-Based Range-Frequency Price Judgments

Section B – Choice Model Parameter Estimates

Section C – Test of Moderation for the Weighting Parameter in Range-Frequency Theory

SECTION A
MEMORY-BASED RANGE-FREQUENCY PRICE JUDGMENTS

Table A1 illustrates the computations for memory-based range (Equation 6), frequency (Equation 7), and range-frequency price judgments (Equation 9) for hypothetical household h and brand j. For illustration purposes, we assume that household h purchased brand j on 15 consecutive weeks. Although we determine ϕ in Equation 9 for each product category empirically, we assume here that ϕ is 10 weeks; thus, occasions 1-10 represent the initialization sample. While we illustrate the calculations of the price judgments in the initialization sample, these values are not included in the estimation sample. In the initialization sample, note that at t = 1 there is one price in the reference price set and range-frequency theory is undefined due to the division by zero. At t = 2, there are two prices in the set, where $.69 is the minimum price and has a rank of 1. The use of ascending or descending ranks is arbitrary but determines the interpretation of the range and frequency values (i.e., whether 0 or 1 is the most attractive price judgment). At t = 4, there are four prices in the set and three prices with the same minimum price of $.69, which receives the rank 2 at t = 4 (the mean rank for ties). In the estimation sample, note that at t = 11, the reference price set used to compute the price judgments comes from occasions 2 through 11 (i.e., ϕ is 10 weeks). At t = 12, the reference price set comes from occasions 3 through 12. For this example, w in equation 9 is assumed to be .5 (i.e., the range and frequency price judgments receive equal weight in computing the range-frequency price judgment).

TABLE A1
MEMEORY-BASED RANGE-FREQUENCY PRICE JUDGMENTS

T	$price_{jht}$	$price_{min,\, jh\phi}$	$price_{max,\, jh\phi}$	$N_{jh\phi}$	$rank_{jht}$	*Range (Eq.6)*	*Freq. (Eq.7)*	$PJ_{RF\,,mem,\, jht}$ *(Eq. 9)*
1	.81	.81	.81	1	1.0	--	--	--
2	.69	.69	.81	2	1.0	.00	.00	.00
3	.69	.69	.81	3	1.5	.00	.25	.13
4	.69	.69	.81	4	2.0	.00	.33	.17
5	.87	.69	.87	5	5.0	1.00	1.00	1.00
6	.87	.69	.87	6	5.5	1.00	.90	.95
7	.81	.69	.87	7	4.5	.67	.58	.63
8	.81	.69	.87	8	5.0	.67	.57	.62
9	.87	.69	.87	9	8.0	1.00	.88	.94
10	.87	.69	.87	10	8.5	1.00	.83	.92
11	.81	.69	.87	10	5.0	.67	.44	.56
12	.49	.49	.87	10	1.0	.00	.00	.00
13	.49	.49	.87	10	1.5	.00	.06	.03
14	.49	.49	.87	10	2.0	.00	.11	.06
15	.75	.49	.87	10	4.0	.68	.33	.51

SECTION B
CHOICE MODEL PARAMETER ESTIMATES

Section B contains the output from the choice models containing the baseline price judgments (Table B1) and the models containing the baseline and range-frequency price judgments (Table B2). Parameter estimates for the models containing only the range-frequency price judgments are provided in Table 2 in the manuscript.

In Tables B1 and B2, b is the estimated mean of the parameter distribution, s is the estimated standard deviation of the parameter distribution, and t is the t-value of the estimate. (---) indicates that the variable was not included in the model. Where applicable, the tables also provide the estimated values of the weighting parameters for loyalty (δ), the baseline memory-based price judgment (α), and the range-frequency price judgment (w).

The variables in the models are denoted by:

$Feature_{jht}$ = 1 if Brand j was featured, 0 otherwise
$Display_{jht}$ = 1 if Brand j was displayed, 0 otherwise
$Loyalty_{jht}$ = Brand loyalty (Equation 2 in the manuscript)
$Brand_j$ = Brand dummy variables (β_{oj} in Equation 1)
$Price_{jht}$ = Price of brand j for household h on purchase occasion t
$PJ_{stim(L),\ jht}$ = Baseline(loss) stimulus-based price judgment (Equation 3 in the manuscript)
$PJ_{stim(G),\ jht}$ = Baseline(gain) stimulus-based price judgment (Equation 3 in the manuscript)
$PJ_{mem,\ jht}$ = Baseline memory-based price judgment (Equation 4 in the manuscript)
$PJ_{RF,\ stim,\ jht}$ = Range-frequency stimulus-based price judgment (Equation 8 in the manuscript)
$PJ_{RF,\ mem,\ jht}$ = Range-frequency memory-based price judgment (Equation 9 in the manuscript)

TABLE B1
PARAMETER ESTIMATES FOR CHOICE MODELS CONTAINING
BASELINE PRICE JUDGMENTS

	S.1 Catsup		*S.1 Tuna*		*S.1 SM*		*M.1 Catsup*		*M.1 Tuna*		*M.1 SM*	
Estimated Mean of the Parameter Distribution												
Variables	*b*	*t*	*b*	*t*	b	*t*	*b*	*t*	*b*	*t*	*b*	*t*
$Feature_{jht}$	3.37	4.26	2.39	10.91	1.34	7.37	2.92	4.03	2.26	9.62	1.11	7.63
$Display_{jht}$	3.64	2.88	1.25	3.47	.66	1.86	3.14	2.83	1.19	3.11	.76	1.90
$Loyalty_{jht}$	2.57	5.42	2.42	10.98	3.47	13.78	2.96	6.31	2.95	13.52	3.89	13.93
Brand 1	3.12	4.09	1.86	6.16	2.01	4.40	1.55	1.86	1.59	5.77	1.23	2.47
Brand 2	1.72	2.69	.87	2.87	2.10	6.10	.77	1.26	.67	2.26	1.48	4.62
Brand 3	2.00	2.59	1.57	5.67	2.26	6.72	.98	1.45	1.24	4.76	1.61	4.70
Brand 4	---	---	---	---	−.01	−.00	---	---	---	---	−5.18	−.19
Brand 5	---	---	---	---	−1.19	−.13	---	---	---	---	−2.55	−.21
$Price_{jht}$	---	---	---	---	---	---	−3.46	−1.69	−8.25	−4.76	−9.06	−6.99
$PJ_{stim(L),\ jht}$	−7.42	−2.44	−16.30	−4.68	−14.69	−8.86	---	---	---	---	---	---
$PJ_{stim(G),\ jht}$	−8.21	−3.93	−16.22	−7.84	−10.25	−7.31	---	---	---	---	---	---
$PJ_{mem,\ jht}$	---	---	---	---	---	---	−4.40	−2.44	−4.98	−4.84	−3.80	−2.89
Standard Deviation of the Parameter Distribution												
Variables	*s*	*t*	*s*	*t*	*s*	*t*	*s*	*t*	*s*	*t*	*s*	*t*
$Feature_{jht}$	1.75	1.31	.55	1.07	.72	2.44	1.98	1.88	.47	.95	.12	.44
$Display_{jht}$	2.97	1.75	1.56	2.19	1.53	2.57	2.73	1.51	1.44	2.77	1.31	2.12
$Loyalty_{jht}$	1.46	1.09	1.84	6.09	1.84	3.25	1.65	1.64	1.71	2.37	2.01	2.26
Brand 1	1.79	1.29	1.95	5.82	2.10	3.07	.80	.61	1.35	3.11	1.37	1.94
Brand 2	2.21	2.00	1.02	4.06	1.89	3.28	1.40	1.06	.66	1.05	1.61	2.38
Brand 3	2.40	1.68	2.01	6.41	1.70	3.01	1.49	1.19	1.54	3.93	1.51	2.46
Brand 4	---	---	---	---	5.85	1.19	---	---	---	---	6.47	.38
Brand 5	---	---	---	---	6.12	.85	---	---	---	---	3.56	.48
$Price_{jht}$	---	---	---	---	---	---	4.58	3.41	6.33	1.97	4.81	1.54
$PJ_{stim(L),\ jht}$	7.27	1.19	10.26	3.47	7.69	2.83	---	---	---	---	---	---
$PJ_{stim(G),\ jht}$	7.75	2.10	9.48	3.28	5.53	1.71	---	---	---	---	---	---
$PJ_{mem,\ jht}$	---	---	---	---	---	---	3.96	1.37	3.98	1.25	4.36	1.31
Weights												
δ	.660		.705		.716		.660		.705		.716	
α	---		---		---		.313		.305		.539	

Notes: The estimated mean (b) and the standard deviation (s) of the parameter distribution are provided for the utility models containing the baseline price judgment in the stimulus-based choice context (S.1) and in the memory-based choice context (M.1) for each product category. The weights are the estimates for δ in Equation 2 and α in Equation 4.

TABLE B2
PARAMETER ESTIMATES FOR CHOICE MODELS CONTAINING BASELINE AND RANGE-FREQUENCY PRICE JUDGMENTS

	S.3 Catsup		*S.3 Tuna*		*S.3 SM*		*M.3 Catsup*		*M.3 Tuna*		*M.3 SM*	
Estimated Mean of the Parameter Distribution												
Variables	*b*	*t*	*b*	*t*	*b*	*t*	*b*	*t*	*b*	*t*	*b*	*t*
$Feature_{jht}$	3.31	4.67	2.48	7.95	1.68	6.33	3.26	4.69	2.30	10.70	1.32	6.16
$Display_{jht}$	3.39	2.67	1.19	2.55	.63	2.18	3.56	2.28	1.18	3.54	.73	1.87
$Loyalty_{jht}$	3.17	5.23	2.31	8.88	3.70	11.46	3.33	6.06	2.99	11.28	4.12	13.83
Brand 1	3.27	3.86	1.88	5.97	2.05	4.04	1.83	1.93	1.63	5.64	1.32	2.37
Brand 2	1.63	2.52	.93	3.18	2.16	6.36	1.06	1.52	.74	2.39	1.60	4.52
Brand 3	2.01	2.57	1.47	4.97	2.32	7.19	1.13	1.40	1.19	4.44	1.70	4.40
Brand 4	---	---	---	---	1.67	.35	---	---	---	---	−3.39	−.18
Brand 5	---	---	---	---	−.88	−.08	---	---	---	---	−.67	−.14
$Price_{jht}$	---	---	---	---	---	---	−4.76	−1.70	−6.36	−4.25	−9.30	−6.15
$PJ_{stim(L),\ jht}$	−5.44	−1.96	−9.88	−1.76	−13.13	−6.15	---	---	---	---	---	---
$PJ_{stim(G),\ jht}$	−8.17	−3.23	−9.44	−3.91	−7.98	−4.51	---	---	---	---	---	---
$PJ_{RF,\ stim,\ jht}$	−1.00	−1.62	−1.58	−3.24	−.83	−2.02	---	---	---	---	---	---
$PJ_{mem,\ jht}$	---	---	---	---	---	---	1.11	.36	−2.76	−2.24	−2.99	−1.89
$PJ_{RF,\ mem,\ jht}$	---	---	---	---	---	---	−2.53	−2.71	−1.87	−3.17	−.45	−1.01
Standard Deviation of the Parameter Distribution												
Variables	*s*	*t*	*s*	*t*	*s*	*t*	*s*	*t*	*s*	*t*	*s*	*t*
$Feature_{jht}$	1.93	1.80	.64	1.30	1.22	3.52	1.66	1.54	.01	.03	.90	2.63
$Display_{jht}$	2.77	1.12	1.70	1.52	.67	1.20	3.16	1.48	1.02	1.59	1.56	2.18
$Loyalty_{jht}$	1.88	1.06	1.93	5.05	2.18	4.37	2.37	1.74	2.02	2.88	2.19	4.66
Brand 1	1.79	1.26	1.83	5.73	2.06	3.50	.58	.25	1.46	1.59	1.95	2.00
Brand 2	1.88	1.76	.74	2.60	1.67	2.65	1.27	.74	.61	.89	1.77	2.98
Brand 3	2.44	1.92	2.04	6.18	1.37	2.67	1.45	.90	1.69	1.75	1.80	2.33
Brand 4	---	---	---	---	4.79	1.11	---	---	---	---	5.01	.47
Brand 5	---	---	---	---	5.87	.76	---	---	---	---	3.76	.85
$Price_{jht}$	---	---	---	---	---	---	5.92	.91	7.15	2.17	4.38	1.65
$PJ_{stim(L),\ jht}$	7.70	1.60	7.72	1.68	7.64	2.35	---	---	---	---	---	---
$PJ_{stim(G),\ jht}$	8.86	1.87	7.03	1.45	5.00	.97	---	---	---	---	---	---
$PJ_{RF,\ stim,\ jht}$	2.11	.93	2.03	1.78	1.79	1.91	---	---	---	---	---	---
$PJ_{mem,\ jht}$	---	---	---	---	---	---	6.89	.82	4.03	1.23	4.51	.84
$PJ_{RF,\ mem,\ jht}$	---	---	---	---	---	---	2.86	.67	2.56	1.00	1.74	1.38
Weights												
δ	.660		.705		.716		.660		.705		.716	
α	---		---		---		.313		.305		.539	
w	.229		.381		.140		.120		.221		.255	

Notes: The estimated mean (b) and standard deviation (s) of the parameter distribution are provided for the utility models containing the baseline and range-frequency price judgments in the stimulus-based choice context (S.3) and in the memory-based choice context (M.3) for each product category. The weights are the estimates for δ in Equation 2, α in Equation 4, and w in Equations 8 (S.3) or 9 (M.3).

SECTION C
TEST OF MODERATION FOR THE WEIGHTING PARAMETER
IN RANGE-FREQUENCY THEORY

We used the following approach to test for moderation of the weighting parameter, *w*, in memory-based range-frequency price judgments (Equation 9). First, we modified *w* as:

$$w = \alpha_1 + \alpha_2 D$$

where D is a 0, 1 dummy variable (e.g., $Trend_{jht}$ in the manuscript). We then added the memory-based range-frequency price judgment to the base utility model (Equation 1) as follows:

$$\begin{aligned} \beta[wR + (1 - w)F] &= \beta F + \beta w(R - F) \\ &= \beta F + \beta(\alpha_1 + \alpha_2 D)(R - F) \\ &= \beta F + \beta\alpha_1(R - F) + \beta\alpha_2 D(R - F) \\ &= \beta F + \theta_1(R - F) + \theta_2 D(R - F) \end{aligned}$$

where β is the coefficient for the range-frequency price judgment, R is the rank price judgment (Equation 6), and F is the frequency price judgment (Equation 7). We then estimated the model by maximum likelihood. The parameters α_1 and α_2 are nonlinear functions of β, θ_1, and θ_2:

$$g = \begin{bmatrix} g_1(\beta, \theta_1, \theta_2) \\ g_2(\beta, \theta_1, \theta_2) \end{bmatrix} = \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{bmatrix} = \begin{bmatrix} \hat{\theta}_1 / \hat{\beta} \\ \hat{\theta}_2 / \hat{\beta} \end{bmatrix}.$$

The asymptotic distribution of a nonlinear function of the maximum likelihood estimator is obtained by the Delta method (See William Greene, *Econometric Analysis, 5th Edition*, 2003, Prentice-Hall, Theorem D.22, page 916). The Jacobian matrix is

$$G = \begin{bmatrix} \partial g_1/\partial\beta & \partial g_1/\partial\theta_1 & \partial g_1/\partial\theta_2 \\ \partial g_2/\partial\beta & \partial g_2/\partial\theta_1 & \partial g_2/\partial\theta_2 \end{bmatrix} = \begin{bmatrix} -\theta_1/\beta^2 & 1/\beta & 1/\beta \\ -\theta_2/\beta^2 & 1/\beta & 1/\beta \end{bmatrix}.$$

Denote the asymptotic covariance matrix of the MLE for β, θ_1, and θ_2 as

$$V = \begin{bmatrix} \text{var}(\beta) & \text{cov}(\beta, \theta_1) & \text{cov}(\beta, \theta_2) \\ \text{cov}(\beta, \theta_1) & \text{var}(\theta_1) & \text{cov}(\theta_1, \theta_2) \\ \text{cov}(\beta, \theta_2) & \text{cov}(\theta_1, \theta_2) & \text{var}(\theta_2) \end{bmatrix}.$$

The asymptotic covariance matrix of g is given by

$$\text{cov}(g) = \text{cov}\begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{bmatrix} = GVG',$$

and the standard errors are the square roots of the diagonal elements of the matrix. These can be used for hypothesis tests that are asymptotically valid. Thus, w differs significantly across groups if the estimate of α_2 is significantly different from 0 using a t-test.