“Speed of Replacement”: Modeling Brand Loyalty Using Last-Move Data

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Web Appendix

Derivation of Equation (11) of the Manuscript

Under our proposed model, the household likelihood function can be simplified as follows.

$$\begin{aligned} L_h &= \mu_{i_h j_h}\left(t_h\right)\exp\left[-\int_0^{u_h}\sum_{k=1}^{J}\mu_{j_h k}\left(t\right)dt\right] \\ &= \mu_{i_h j_h}\left(t_h\right)\exp\left[-\sum_{k=1}^{J}\int_0^{u_h}\mu_{j_h k}\left(t\right)dt\right] \\ &= \left[\alpha_{i_h}\left(h_{j_h}\left(t_h\right)+\delta\right)\mathrm{I}\left(i_h=j_h\right)+\left(1-\alpha_{i_h}\right)h_{j_h}\left(t_h\right)\right]\exp\left[-\sum_{k=1}^{J}\int_0^{u_h}\left[\alpha_{j_h}\left(h_k\left(t\right)+\delta\right)\mathrm{I}\left(j_h=k\right)+\left(1-\alpha_{j_h}\right)h_k\left(t\right)\right]dt\right], \end{aligned}$$

where $I(i_h = j_h)$ is an indicator function that take the value 1 when $i_h = j_h$, and $I(j_h = k)$ is an indicator function that take the value 1 when $j_h = k$.

Below, we derive the sample likelihood function for the special cases that arise out of values taken by $I(i_h = j_h)$ and $I(j_h = k)$.

1. When $i_h = j_h$ and $j_h = k$,

$$
\begin{aligned}
L &= \left[h_{j_h}\left(t_h\right)+\alpha_{i_h}\delta\right]\exp\left[-\sum_{k=1}^{J}\int_0^{u_h}\left[h_k\left(t\right)+\alpha_{j_h}\delta\right]dt\right] \\
&= \left[h_{j_h}\left(t_h\right)+\alpha_{i_h}\delta\right]\exp\left[-\sum_{k=1}^{J}\left[\alpha_{j_h}\delta u_h+\int_0^{u_h}h_k\left(t\right)dt\right]\right] \\
&= \left[h_{j_h}\left(t_h\right)+\alpha_{i_h}\delta\right]\exp\left(-J\alpha_{j_h}\delta u_h\right)\exp\left[-\sum_{k=1}^{J}\int_0^{u_h}h_k\left(t\right)dt\right] \\
&= \left[h_{j_h}\left(t_h\right)+\alpha_{i_h}\delta\right]\exp\left(-J\alpha_{j_h}\delta u_h\right)\prod_{k=1}^{J}\exp\left(-\int_0^{u_h}h_k\left(t\right)dt\right) \\
&= \left[h_{j_h}\left(t_h\right)+\alpha_{i_h}\delta\right]\exp\left(-J\alpha_{j_h}\delta u_h\right)\prod_{k=1}^{J}S_k\left(u_h\right)
\end{aligned}
$$

When $i_h = j_h$ and $j_h \neq k$,

$$L = \left[h_{j_h}(t_h) + \alpha_{i_h}\delta \right] \exp\left[-\sum_{k=1}^{J} \int_0^{u_h} \left[\left(1-\alpha_{j_h}\right) h_k(t) \right] dt \right]$$
$$= \left[h_{j_h}(t_h) + \alpha_{i_h}\delta \right] \prod_{k=1}^{J} \exp\left(-\int_0^{u_h} \left(1-\alpha_{j_h}\right) h_k(t)\, dt \right)$$
$$= \left[h_{j_h}(t_h) + \alpha_{i_h}\delta \right] \prod_{k=1}^{J} \left[S_k(u_h) \right]^{1-\alpha_{j_h}}$$

2. When $i_h \neq j_h$ and $j_h = k$,

$$L = \left[\left(1-\alpha_{i_h}\right) h_{j_h}(t_h) \right] \exp\left(-J\alpha_{j_h}\delta u_h \right) \prod_{k=1}^{J} S_k(u_h)$$

3. When $i_h \neq j_h$ and $j_h = k$,

$$L = \left[\left(1-\alpha_{i_h}\right) h_{j_h}(t_h) \right] \prod_{k=1}^{J} \left[S_k(u_h) \right]^{\left(1-\alpha_{i_h}\right)}$$

Under the log-logistic distribution,

$$h_k(u_h) = \frac{\gamma_k \lambda_k \left(\gamma_k u_h\right)^{\lambda_k - 1}}{1 + \left(\gamma_k u_h\right)^{\lambda_k}},$$
$$S_k(u_h) = \frac{1}{1 + \left(\gamma_k u_h\right)^{\lambda_k}},$$
$$(\gamma_k, \lambda_k > 0)$$

Next, we summarize results of additional empirical analyses that we performed in response to issues raised by reviewers.

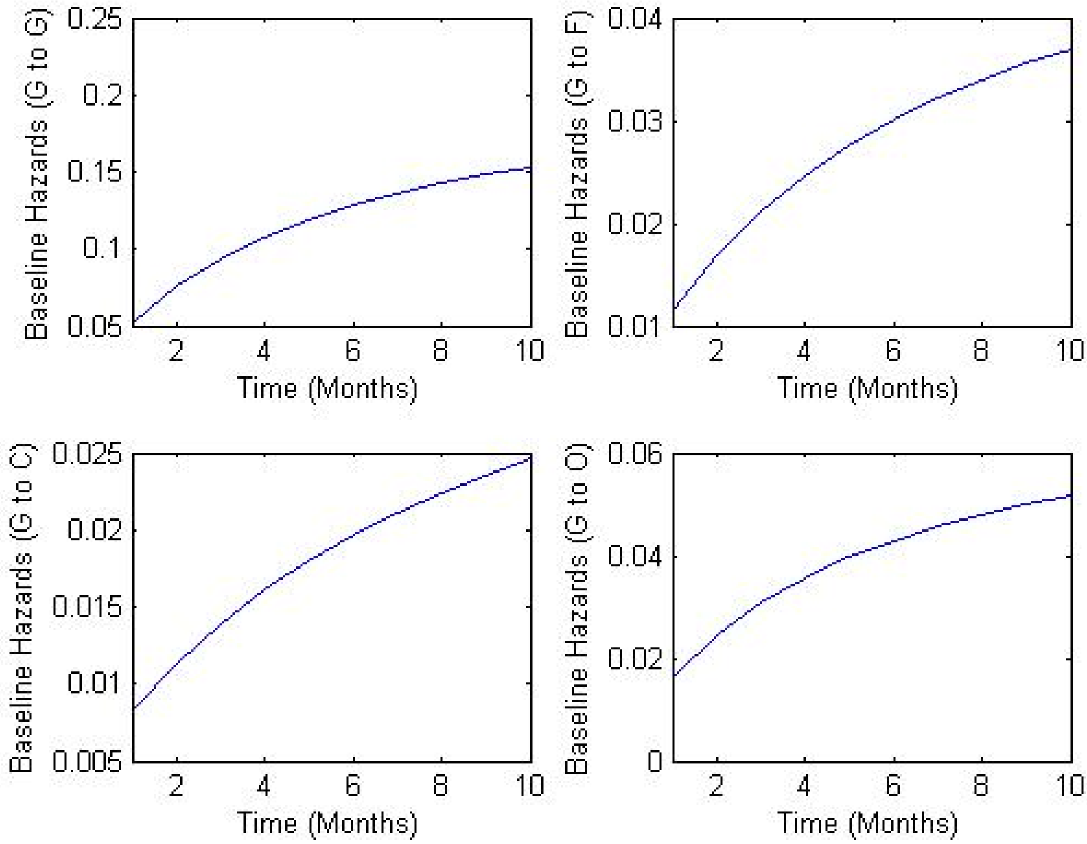


FIGURE A1: ESTIMATED ERLANG-2 TRANSITION HAZARDS ASSOCIATED WITH SWITCHING AWAY FROM GM (i=GM → j)

TABLE A1
Results from the Full Version of Our Proposed Model with Erlang-2 Baseline Hazard

Parameters		Estimates*
Brand Loyalties (α_i)	GM	0.61 (0.04)
	Ford	0.81 (0.03)
	Chrysler	0.54 (0.05)
	Other	0.78 (0.03)
Demographic Effects (β)	Marital Status	-0.04 (0.02)
	Income	0.02 (0.01)
	Education	-0.01 (0.005)
	Satisfaction	0.04 (0.01)
Baseline Hazard Parameters (γ_j)	GM	0.22 (0.01)
	Ford	0.15 (0.01)
	Chrysler	0.11 (0.01)
	Other	0.20 (0.01)
Increase in Hazard Rate due to Loyalty (δ)		0.01 (0.00)
Effect of modal replacement time (τ)		0.01 (0.00)

*Standard errors within parentheses. All estimates are significant at the 0.05 level.

Number of Households = 870, Number of Parameters = 14
LL = -3263.40, AIC = 6554.80, BIC = 6621.56

TABLE A2
Fit Results for Our Proposed Model with Unobserved Heterogeneity

Heterogeneous (2-Segment) Log-Logistic Version of Our Proposed Model

LL = -2782.46

Parameters = 27

AIC = 5618.32

SBC = 5747.07

Heterogeneous (2-Segment) Erlang-2 Version of Our Proposed Model

LL = -3252.73

Parameters = 19

AIC = 6543.44

SBC = 6634.04

FIGURE A2: ESTIMATED LOG-LOGISTIC TRANSITION HAZARDS ASSOCIATED WITH SWITCHING AWAY FROM GM ($i=GM \rightarrow j$) OBTAINED USING THE NEW 2002 DATASET

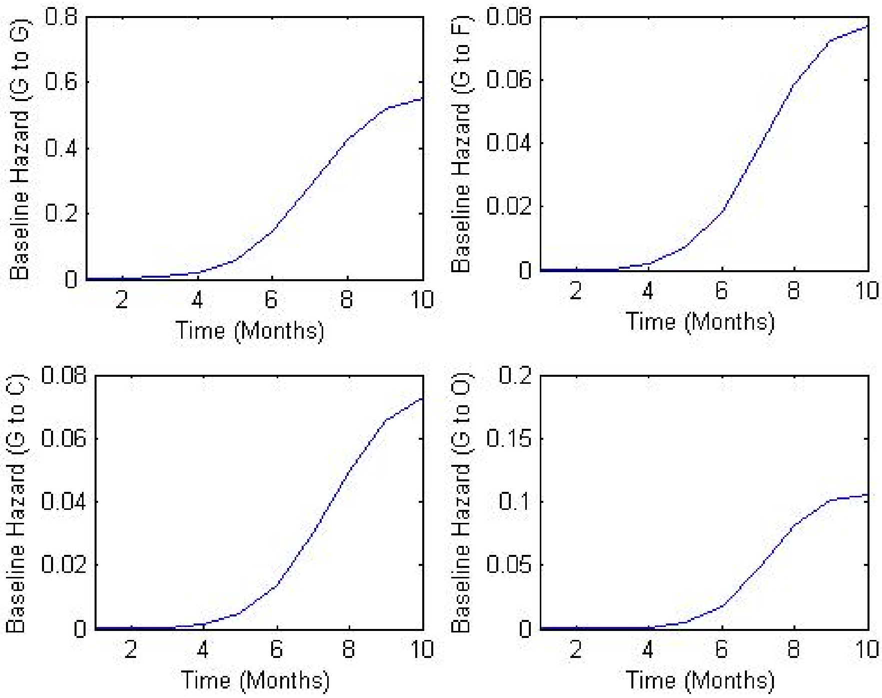
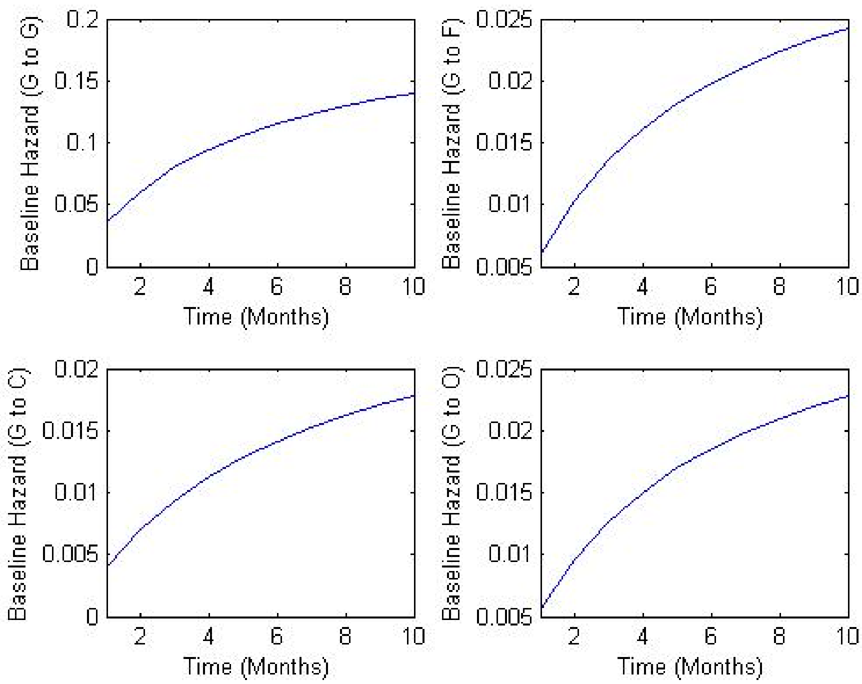


FIGURE A3: ESTIMATED ERLANG-2 TRANSITION HAZARDS ASSOCIATED WITH SWITCHING AWAY FROM GM (i=GM → j) OBTAINED USING THE NEW 2002 DATASET



Next, we summarize the empirical results obtained using the new 2002 survey dataset. Tables A3-A12 correspond to Tables 1-10 in the manuscript, respectively.

TABLE A3

Observed Brand Switching Matrix
(Row=previously owned brand, Column=currently owned brand)

	GM	Ford	Chrysler	Other	Total
GM	117 (0.71)	10 (0.06)	14 (0.09)	23 (0.14)	**164**
Ford	17 (0.13)	84 (0.63)	11 (0.08)	21 (0.16)	**133**
Chrysler	13 (0.18)	14 (0.20)	31 (0.44)	13 (0.18)	**71**
Other	7 (0.10)	9 (0.13)	3 (0.04)	52 (0.73)	**71**
Total	**154**	**117**	**59**	**109**	**439**

TABLE A4
Descriptive Statistics

(Number of respondents in sample: 439)

	Mean	Std Dev	Max	Min
Elapsed Time (months)	7.54	2.53	36	1
Satisfaction	5.64	1.51	7	1

TABLE A5
Fit Statistics
AIC = -2*LL + 2 (# parameters)
SBC = -2*LL + [ln (# households)] * [# parameters]
Number of households = 439

	LL	**AIC**	**SBC**	**# param.**
Full Model	-1317.66	2667.32	2732.67	16
Stationary Version	-1761.06	3542.12	3583.01	10
Stationary Version w/o Covariates	-1765.51	3547.02	3583.78	9
Stationary Version w/o Covariates & Loyalty Structure	-1757.64	3547.28	3612.63	16

TABLE A6
Results from the Full Version of Our Proposed Model

Parameters		Estimates*
Brand Loyalties (α_i)	GM	0.87 (0.02)
	Ford	0.81 (0.03)
	Chrysler	0.68 (0.05)
	Other	0.85 (0.03)
Covariates (β)	Satisfaction	0.03 (0.01)
Baseline Hazard Rates – Shape Parameters (γ_j)	GM	0.12 (0.00)
	Ford	0.12 (0.00)
	Chrysler	0.12 (0.00)
	Other	0.13 (0.00)
Baseline Hazard Rates – Scale Parameters (λ_j)	GM	6.73 (0.51)
	Ford	7.04 (0.59)
	Chrysler	7.08 (0.77)
	Other	8.87 (0.71)
Increase in Hazard Rate due to Loyalty (δ)		0.002 (0.001)
Effect of model replacement time (τ)		0.04 (0.01)
Error standard deviation (σ)		0.002 (0.001)

*Standard errors within parentheses. All estimates are significant at the 0.05 level.

Number of Households = 439, Number of Parameters = 16
LL = -1317.66, AIC = 2667.32, BIC = 2732.67

TABLE A7
Results from the Stationary Version of Our Proposed Model

Parameters		Estimates*
Brand Loyalties (α_i)	GM	0.83 (0.03)
	Ford	0.77 (0.04)
	Chrysler	0.68 (0.06)
	Other	0.89 (0.02)
Demographic Effects (β)	Satisfaction	0.03 (0.01)
Baseline Hazard Rates (μ_j)	GM	0.10 (0.01)
	Ford	0.09 (0.01)
	Chrysler	0.06 (0.01)
	Other	0.08 (0.01)
Increase in Hazard Rate due to Loyalty (δ)		0.006 (0.002)

*Standard errors within parentheses.

Number of Households = 439, Number of Parameters = 14
LL = -1761.06, AIC = 3542.12, BIC = 3583.01

TABLE A8
Results from the Stationary Version of Our Proposed Model without Covariates

Parameters		Estimates*
Brand Loyalties (α_i)	GM	0.83 (0.03)
	Ford	0.77 (0.04)
	Chrysler	0.68 (0.06)
	Other	0.90 (0.02)
Baseline Hazard Rates (μ_j)	GM	0.10 (0.01)
	Ford	0.09 (0.01)
	Chrysler	0.06 (0.01)
	Other	0.08 (0.01)
Increase in Hazard Rate due to Loyalty (δ)		0.006 (0.002)

*Standard errors within parentheses.

Number of Households = 439, Number of Parameters = 9
LL = -1765.51, AIC = 3547.02, BIC = 3583.78

TABLE A9
Results from the Stationary Version of Our Proposed Model without Demographics and without the Parsimonious Brand Loyalty Structure

	GM	Ford	Chrysler	Other
GM	0.10 (0.01)	0.01 (0.00)	0.01 (0.00)	0.02 (0.00)
Ford	0.02 (0.00)	0.09 (0.01)	0.01 (0.00)	0.02 (0.01)
Chrysler	0.03 (0.01)	0.03 (0.01)	0.07 (0.01)	0.03 (0.01)
Other	0.01 (0.00)	0.01 (0.00)	0.00 (0.00)	0.07 (0.01)

*Standard errors within parentheses.

Number of Households = 439, Number of Parameters = 16
LL = -1757.64, AIC = 3547.28, BIC = 3612.63

TABLE A10
Transition Rate Matrix Implied by the Full Version of our Proposed Model

	GM	Ford	Chrysler	Other
GM	0.10	0.01	0.02	0.02
Ford	0.02	0.09	0.01	0.02
Chrysler	0.02	0.03	0.07	0.03
Other	0.01	0.01	0.01	0.07

TABLE A11
Results from the Colombo and Morrison (1989) Model

Parameters		Estimates*
Brand Loyalties (α_i)	GM	0.61 (0.05)
	Ford	0.52 (0.06)
	Chrysler	0.31 (0.08)
	Other	0.60 (0.08)
Zero-order Brand Choice Probabilities (μ_i)	GM	0.26 (0.05)
	Ford	0.23 (0.07)
	Chrysler	0.18 (0.08)
	Other	0.32

*Standard errors within parentheses. All estimates are significant at the 0.05 level.

Number of Observations = 16, Number of Parameters = 7
LL = - 442.68, AIC = 899.36, BIC =904.77

TABLE A12
Estimated Transition Probabilities under Our Model versus Colombo and Morrison (1989) Model (within parentheses)

	GM	Ford	Chrysler	Others
GM	0.71 (0.71)	0.06 (0.09)	0.09 (0.07)	0.14 (0.13)
Ford	0.13 (0.13)	0.63 (0.63)	0.08 (0.09)	0.16 (0.16)
Chrysler	0.18 (0.18)	0.20 (0.16)	0.44 (0.44)	0.18 (0.22)
Others	0.10 (0.10)	0.13 (0.09)	0.04 (0.07)	0.73 (0.73)

RMSE of Our Proposed Model = 0.0007
RMSE of Colombo and Morrison = 0.0829