WEB APPENDIX

Do Switching Costs Make Markets Less Competitive?

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Web Appendix A: Numerical Solution to the Dynamic Program

We use numerical methods to solve for the equilibrium of the pricing game. We first discretize each axis of the state space using a finite number of points, $0 < x_{i0} < x_{i1} < ... < x_{iL} = 1$. We then form a grid representing the whole state space from the Cartesian product of these points. For each point in the grid, we store the value and policy functions of each competitor in the computer memory. For states outside the grid, we calculate the value and policy functions using bilinear interpolation. To solve for the equilibrium, we employ the following algorithm, which is an adaptation of policy iteration applied to the case of the games: start with some initial guess of the strategy profile, $\sigma^0 = (\sigma_1^0, ..., \sigma_J^0)$, and then proceed along the following steps:

1. For the strategy profile σ^n, calculate the corresponding value functions for each of the J firms. These value function are defined by the Bellman equation (5), where the right hand side of the Bellman equation is not maximized, but instead evaluated using the current strategy profile σ^n.
2. If $n > 0$, check whether the value functions and policy functions satisfy the convergence criteria, $\left\| V_j^n - V_j^{n-1} \right\| < \varepsilon_V$ and $\left\| \sigma_j^n - \sigma_j^{n-1} \right\| < \varepsilon_\sigma$ for all firms j. If so, stop.

Update each firm's strategy using the Bellman equation. In contrast to step 1, the maximization on the right hand side is now carried out. Denote the resulting new policies and value functions by σ_j^{n+1} and V_j^{n+1}, and return to step 1.

Web Appendix B1: Existence of Equilibrium in the Simple Model

The existence of a Markov perfect equilibrium in our model follows from arguments given in Whitt (1980) and Doraszelski and Satterthwaite (2005). In order to show that the equilibrium is in pure strategies, we need to show that the best-reply correspondence is single-valued. Our strategy is to show that in the case of one consumer with logit demand, the right-hand side of the Bellman equation is strictly quasi-concave, and hence has a unique maximizer. This strategy has been employed previously by Besanko et al. (2007).

The Bellman equation in the simple model is given by

$$V_j(s) = \max_{p_j \geq 0} \left\{ \pi_j(s,p) + \beta\left(\sum\nolimits_{k=1}^{J} P_k(s,p) V_j(k) + P_0(s,p) V_j(s) \right) \right\} \qquad \forall s \in X.$$

Denote the right-hand side of this functional equation by $\Psi_j(s, p_j, p_{-j}),$ such that

$$V_j(s) = \max_{p_j \geq 0} \Psi_j(s, p_j, p_{-j}), \qquad \forall s \in X.$$

This maximization problem has the following first-order condition:

$$\begin{aligned} \frac{\partial \Psi_j}{\partial p_j} &= \alpha P_j(1-P_j)(p_j - c_j) + P_j + \beta\left(\sum\nolimits_{k=1}^{J} (-\alpha) P_j P_k V_j(k) + \alpha P_j V_j(j) - \alpha P_0 P_j V_j(s) \right) \\ &= \alpha P_j \left(-\Psi_j + (p_j - c_j) + \frac{1}{\alpha} + \beta V_j(j) \right). \end{aligned}$$

Here,P_j s shorthand for $P_j(s, p_j, p_{-j}).$ Evaluating the second-order condition at a price where $\partial \Psi_j / \partial p_j = 0,$ we find that

$$\begin{aligned}\frac{\partial^2\Psi_j}{\partial p_j^2} &= \alpha^2 P_j(1-P_j)\left(-\Psi_j + (p_j - c_j) + \frac{1}{\alpha} + \beta V_j(j)\right) + \alpha P_j\left(-\frac{\partial\Psi_j}{\partial p_j} + 1\right) \\ &= \alpha(1-P_j)\frac{\partial\Psi_j}{\partial p_j} + \alpha P_j\left(-\frac{\partial\Psi_j}{\partial p_j} + 1\right) \\ &= \alpha P_j < 0.\end{aligned}$$

Hence, Ψ_j is strictly quasi-concave in p_j, and hence it follows that there is a unique price that maximizes the right-hand side of the Bellman equation for any state s and price profile $p_{-j} = \sigma_{-j}(s)$.

Web Appendix B2: Simplified Model Without Logit Error

If we remove the random utility component in (1), ε_{jt}, the equilibrium can be characterized analytically. We focus on the case of symmetry across players, where all firms have the same utility intercepts (i.e. no vertical differentiation) and costs. Removing ε_{jt} is analogous to eliminating the (horizontal) product differentiation and, hence, we are left with a model of homogeneous products. We assume that the common utility intercept, $\delta > c \geq 0$.

Proposition. *Let* υ *be such that* $0 \le \upsilon \le (1-\beta)\gamma$ *and* $c + \upsilon \le \delta + \gamma$. *Then under the assumptions stated above there is a symmetric Markov perfect equilibrium with pricing strategies* $\sigma_j^*(j) = c + \upsilon$ *and* $\sigma_j^*(k) = c + \upsilon - \gamma$ *for all* $k \in X,\ k \ne j$.

Proof. j denotes the product to which the customer is loyal and k denotes any other product. Because $p_j = c + \upsilon = p_k + \gamma$, the customer's utility index is the same for all products. Therefore, by assumption she will not switch from product j to k, and because $0 \le \delta + \gamma - (c + \upsilon)$, she will not choose the outside option. The value from this strategy is $V_j(j) = (1-\beta)^{-1}\upsilon$ and $V_j(k) = 0$. In order to assess whether the proposed strategies constitute a best response for each player, we only need to consider one-period deviations. If firm *j* reduces its price, it will reduce its current-period profit and leave its future value unchanged. If firm *j* raises its price, it will loose its loyal customer and receive a payoff of zero now and in future. Hence, $p_j = c + \upsilon$ is a best response to p_k. Competitor *k* needs to offer a price

$p_k = c + \upsilon - \gamma - \varepsilon,\ \ \varepsilon > 0$, in order to acquire the customer. Because $\upsilon \leq (1-\beta)\gamma$, the present value from this one-period deviation is negative:

$$\upsilon - \gamma - \varepsilon + \beta \frac{\upsilon}{1-\beta} = \frac{\upsilon}{1-\beta} - \gamma - \varepsilon < 0.$$

Alternatively, firm *k* cannot improve on its current outcome by raising its price, and hence,

$p_k = c + \upsilon - \gamma$ is a best response to p_j.

Web Appendix C: Comparative Statics Holding the Market Size Constant

In order to eliminate the difference between the state dependence and the pure switching cost models, which is due to the differential impact on the outside good market share under changes in γ we define the outside good intercept as a function of the switching cost. If $\gamma = 0$, we choose some arbitrary intercept, such as $\delta_0 = 0$. We denote the resulting equilibrium prices in state s by $p^0(s)$, and let the corresponding outside good share be P_0^0. For $\gamma > 0$, choose δ_0 such that $P_0(s, p^0(s); \delta_0) = P_0^0$, and note that due to symmetry, the left-hand side of this equation and hence the choice of δ_0 is the same for either state, $s = 1,2$. Under this choice of δ_0, if the firms do not change their prices compared to the case without switching costs, the outside good market share at any $\gamma > 0$ will remain constant at P_0^0, its level under $\gamma = 0$. However, even though the total market size does not change, the customer is more likely to purchase the good to which she is loyal to for larger values of γ. Note that this technique of defining the outside good intercept as a function of the switching cost eliminates any difference between the state-dependent and the pure switching cost model. In particular, at any given price vector and δ_0 as defined above, the predicted demand from either model formulation is identical.

Web Appendix D: Forward-Looking Consumers in the Simple Model

We now extend the model to allow for forward-looking consumers who anticipate the consequences of becoming loyal to product *j*. In general, the presence of forward-looking consumers can complicate the computation of an equilibrium. For example, Anderson, Kumar and Rajiv (2004) show the equilibrium proposed by Padilla (1995) does not in fact constitute a Markov perfect equilibrium under forward-looking consumer behavior.

As before, the current-period utility from choosing product *j* is $U_j = U(j,s,p) + \lambda\varepsilon_j$. But, now consumers maximize the PDV of current and future utilities. For simplicity, we assume that consumers discount future utilities at the same rate as firms, β. Define the state transition function $s' = \phi(s,j) = j$ if $j \neq 0$ and $s' = \phi(s,0) = s$. The value function of the consumer given state *s* and idiosyncratic utility draws $\varepsilon = (\varepsilon_0, \ldots, \varepsilon_J)$ is

$$\upsilon(s,\varepsilon) = \max_{j=0,\ldots,J} \left\{ U(s,\sigma(s),j) + \lambda\varepsilon_j + \beta \int \upsilon(\phi(s,j),\varepsilon') f(\varepsilon') d\varepsilon' \right\}. \qquad \text{(C.1)}$$

Note that this value function depends on the consumer's expectation that the firms choose prices according to $p_j = \sigma_j(s)$. Following arguments given in Rust (1987), the consumer's decision problem can be reformulated in the following way. Let the expected future value from choosing alternative *j* in state *s* be

$$W(s,j) = \int \max_{k=0,\ldots,J} \left\{ U(s',\sigma(s'),k) + \lambda\varepsilon_k + \beta W(s',k) \right\} f\left(\varepsilon\right) d\varepsilon,$$

where $s' = \phi(s,j)$. Since ε has the Type I extreme value distribution, $W(s,j)$ has the closed form expression

$$W\left(s,j\right)=\lambda\left(\gamma+\log\left[\sum_{k=0}^{J}\exp\left(\frac{1}{\lambda}\left(U(s',\sigma(s'),k)+\beta W(s',k)\right)\right)\right]\right). \tag{C.2}$$

Here, $\gamma \approx 0.57722$ is Euler's constant. The consumer then chooses the alternative *j=0, ..., J* that yields the highest utility index

$$U(s,\sigma(s),j)+\beta W(s,\phi(s,j))+\lambda\varepsilon_j .$$

Conditional on the consumer's choice behavior, which is now also described by the consumer's value function, *W,* the firm's problem remains the same under forward-looking consumer behavior. A Markov perfect equilibrium now consists of pricing strategies and value functions for each firm *j* and the consumer's consumption strategy, which is fully described by the value function *W,* such that (i) each firm's pricing strategy is optimal given the consumer's strategy and given the competitors' strategies, and (ii) given the firms' pricing strategies, the consumers value function satisfies equation (C.2).

In Section 2, we explored the predictions of the simple model for the symmetric case with a symmetric equilibrium. In this case, myopic and forward-looking consumer behavior is identical. This can be seen from equation (C.2): *W* actually depends only on $s'=\phi(s,j)$, the product that the consumer is loyal to in the next period. Due to symmetry, the identity of this product does not matter. Therefore, *W* is exactly the same for all $s'\in X$, and therefore adds the same constant to each utility index. Thus, the choice probabilities are not affected by the presence of *W.* Therefore, our pricing results are robust to a forward-looking consumer.

Web Appendix E: Overlapping Generations Version of the Simple Model

We now develop an OLG version of our simple state dependence model and examine the robustness of our previous finding that switching costs can lower equilibrium prices. In each period, a new customer is born. The customer lives for two periods and, hence, the market always consists of a “young” and an “old” customer. A customer can be loyal to one of the *J* products, or she can be unattached, i.e., loyal to the outside alternative. If a customer is loyal to the outside alternative, she does not incur a switching cost for any product choice. Otherwise, her demand is as in the model analyzed before. When the young customer is born, she is unattached. If she chooses the outside alternative, she stays unattached in the next period, when she is old. Otherwise, if she buys product *j* she becomes loyal to *j*. The state of the market is now described by $s_t \in \{0,1,...,J\}$, the choice that the currently old customer made in the previous period, *t-1*.

Table D.1 shows the average transaction prices paid by the young and the old customer for different switching cost levels. The model was solved with forward-looking consumers. Due to lock-in, the old customer always pays a higher average price than the young customer. Unless switching cost levels are sufficiently large, however, both the young and the old customer pay a lower price, on average, than in the case without switching costs. The young customer, in particular, generally pays a lower price. Thus, our main conclusion that switching costs do not necessarily lead to higher prices is robust to a different model formulation as well as a wide range of parameter values.

Table D.1

Equilibrium prices in the OLG model

Switching Cost	p^a_{young}	p^a_{old}
0.00	1.81	1.81
0.25	1.78	1.78
0.50	1.76	1.76
0.75	1.74	1.75
1.00	1.71	1.74
1.25	1.69	1.74
1.50	1.66	1.74
1.75	1.63	1.74
2.00	1.59	1.75
3.00	1.41	1.80
4.00	1.19	1.83
5.00	1.01	1.85
6.00	0.90	1.89
7.00	0.83	1.91
8.00	0.81	1.92

References are available from the authors on request.