Designing Sales Contests: Does the Prize Structure Matter?

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WEB APPENDIX

IMPORTANT FEATURES OF THE MARKETING THEORY OF SALES CONTESTS

Salesperson's Behavior

Please refer to the model setup introduced in the paper. Given the assumptions, each salesperson evaluates the tradeoff between expending effort and the probability of attaining monetary rewards given the prize structure. The expected utility of the salesperson is

(1) $$EU(R_i, e_i) = \sum_{j=1}^{N} \frac{P_j^{\alpha}}{\alpha} \times \Pr(R_i = P_j) - c(e_i).$$

The first term in Equation 1 denotes the expected reward for the salesperson, which is obtained by multiplying the value of the prize for the jth-rank with the probability of attaining that rank, summed across the N possible ranks.

Each salesperson chooses the optimal level of effort by maximizing their expected utility given in Equation 1. Further using the assumption that all salespeople have identical and rational belief structures, so that all salespeople will expend e^*, the probability that a salesperson attains the jth rank is

(2) $$\Pr(R_i = P_j) = \int_{\varepsilon_i} \binom{N-1}{j-1} \times [1 - F(y)]^{j-1} \times F^{N-j}(y) \times f(\varepsilon_i) d\varepsilon_i,$$

where $y = h(e_i) - h(e^*) + \varepsilon_i$, while F and f are the cumulative probability and probability densities of ε_i respectively. The symmetric Nash-equilibrium effort for a salesperson is given by the solution to the following first-order condition:

(3) $$\sum_{j=1}^{N} \frac{P_j^{\alpha}}{\alpha} \frac{\partial \text{Prob}(R_i = P_j)}{\partial e_i}(e_i = e^*) - c(e^*) \equiv 0.$$

When sales outcomes follow the logistic density and $h(e_i) = e_i$, Kalra and Shi (2001) showed that the marginal probability of obtaining the prize associated with the j^{th} rank is equal to $(N-2j+1)/[\beta N(N+1)]$. Using the common functional specification of $c(e_i) = ke_i^2$ (Kalra and Shi 2001; Bull, Schotter and Weigelt 1987), the closed-form equilibrium effort level of each salesperson for a given prize structure is

(4) $$e^* = \sum_{j=1}^{N} \frac{P_j^{\alpha}(N-2j+1)}{2\alpha k\beta N(N+1)}.$$

Manager's Decision

The manager chooses the prize structure that maximizes the firm's expected profits. Letting m be the firm's margin on every sales dollar, the manager solves the following constrained maximization problem:

(5) $$\max_{\{P_1, P_2, \ldots, P_N\}} E\pi = m \times N \times s(e^*) - B$$

(6) $$\text{s.t.} \quad e^* = \sum_{j=1}^{N} \frac{P_j^{\alpha}(N-2j+1)}{2\alpha k\beta N(N+1)};$$

(7) $$\frac{1}{N}\sum_{j=1}^{N} \frac{P_j^{\alpha}}{\alpha} - k(e^*)^2 \geq u_0;$$

(8) $$\sum_{j=1}^{N} P_j = B; \text{ and}$$

(9) $$P_j \geq 0.$$

The four constraints in Equations 6 to 9 are interpreted as follows: Equation 6 says that the manager is incorporating the fact that salespeople will choose the symmetric Nash-equilibrium effort level given a prize structure when designing the optimal contest. Equation 7 says that the prize structure must be designed so that the salesperson's expected utility from participating in the contest is only as high as some level of utility, u_0, which is typically influenced by other selling tasks and incentive structures. Equations 8 and 9 limit the total prize value to the budget, B, and the prize values to be nonnegative, respectively.

ANALYZING THE EFFECT OF HETEROGENEITY ON SALES CONTEST DESIGN

We discuss how the incentive effect of prize spreads on effort differs when salespeople are heterogeneous in their sales productivity, relative to the case when they are homogeneous. To do so, we compare the results of tournament models in the economics literature that allow for heterogeneity with a simplified version of the model (which assumes homogeneity) we present in the paper. We show that while economic theory still predicts a positive marginal effect of prize spreads on effort, this effect is weaker when salespeople are heterogeneous in sales abilities or territorial advantages. This insight suggests that the positive incentive effect from moving from a Grouped-Winners contest to a Rank-Winners contest through increasing the spreads between prizes is smaller when sales people differ in their sales productivity.

We analyze a simplified version of the model presented in the paper. Consider a two-contestant model with risk-neutral salespeople. The results do not change qualitatively if salespeople are risk averse. Let the prizes corresponding to the two ranks be P_1 and P_2, where $P_1 > P_2 \geq 0$. The utility of salesperson i (i = 1, 2) is $U_i(P_i, e_i) = P_i - ke_i^2$, where ke_i^2 is the cost-of-effort function. Sales output is given by $s(e_i) = e_i + \varepsilon_i$, and for simplicity ε_i is assumed to be distributed independently and uniformly over the interval [-q, +q], where q > 0. In this case of homogeneous salespeople, it can be shown that the symmetric Nash equilibrium sales effort is given by $e^* = (P_1 - P_2) / 4kq$ (see Bull, Schotter and Weigelt 1987 for a detailed derivation). This expression is also shown in Table W1.

Next, we consider the results of tournament models that allow for heterogeneity in sales productivity. There are two ways to capture heterogeneity in sales productivity. First, one can allow the cost of effort to be different across salespeople. This type of contest is also known as an "uneven contest". In our analysis, the only change we make to the basic model above is to assume that the cost-of-effort function for the less productive salesperson is $2ke_i^2$. Second, one can capture differences in territorial advantages by adding a term $a > 0$ (with $a < 2q$) to the sales output of the more productive salesperson, so that it becomes $s(e_i) = e_i + a + \varepsilon_i$. This type of contest is also known as an "unfair contest". Keeping all the other assumptions identical to the model setup for the homogeneous case above, the Nash equilibrium effort levels for salespeople in the uneven and unfair contests can be derived and are shown in Table W1. The detailed derivations of the equilibrium effort levels for the uneven and unfair contests are found in Bull, Schotter and Weigelt (1987) and Weigelt, Dukerich and Schotter (1989) respectively. Note that in the unfair contest, both salespeople expend the same level of effort even though their sales productivity is different.

To examine the incentive effect of prize spreads on effort, we take the derivative of effort with respect to the prize spread $(P_1 - P_2)$, that is, we derive $\partial e^* / \partial(P_1 - P_2)$. The expressions for $\partial e^* / \partial(P_1 - P_2)$ for the homogeneous, uneven and unfair contests are shown in the last column of Table W1. We also note that $\partial e^* / \partial(P_1 - P_2) > 0$ in all three cases, so that the incentive effect is always predicted to be positive. Next, we compare the relative strengths of the incentive effect for the homogeneous contest to each of the heterogeneous contests. We find that $\partial e^* / \partial(P_1 - P_2)$

is always stronger in the homogeneous contest, so that increasing the prize spread between ranks motivates salespeople more compared to the cases where the sales productivity between the contestants is different. The proofs of these results are shown below.

TABLE W1

EQUILIBRIUM EFFORT AND THE EFFECT OF PRIZE SPREADS ON EFFORT

	Equilibrium Effort (e^*)	Effect of Spreads on Effort $\left(\partial e^* / \partial (P_1 - P_2)\right)$
Homogeneous Contest	$\frac{P_1 - P_2}{4kq}$	$\frac{1}{4kq}$
Uneven Contest		
More productive salesperson	$\frac{4q(P_1 - P_2)}{16kq^2 + P_1 - P_2}$	$\frac{64kq^3}{(16kq^2 + P_1 - P_2)^2}$
Less productive salesperson	$\frac{2q(P_1 - P_2)}{16kq^2 + P_1 - P_2}$	$\frac{32kq^3}{(16kq^2 + P_1 - P_2)^2}$
Unfair Contest	$\left[\frac{1}{2q} - \frac{a}{4q^2}\right]\frac{P_1 - P_2}{2k}$	$\frac{1}{4kq} - \frac{a}{8kq^2}$

Result 1: $\frac{\partial e^*_{Homogeneous}}{\partial (P_1 - P_2)} > \frac{\partial e^*_{MoreProductive}}{\partial (P_1 - P_2)} > \frac{\partial e^*_{LessProductive}}{\partial (P_1 - P_2)}$.

Proof: We only need to show that the first inequality holds. To begin, we have

$$\frac{1}{4kq} > \frac{64kq^3}{(16kq^2 + P_1 - P_2)^2}$$

$$\Rightarrow (16kq^2 + P_1 - P_2)^2 > 256k^2q^4$$

$$\Rightarrow 16kq^2 + P_1 - P_2 > 16kq^2$$

$$\Rightarrow P_1 - P_2 > 0$$ (which is assumed in the model).

Result 2: $\frac{\partial e^*_{\text{Homogeneous}}}{\partial (P_1 - P_2)} > \frac{\partial e^*_{\text{Unfair}}}{\partial (P_1 - P_2)}$.

Proof: $\frac{1}{4kq} > \frac{1}{4kq} - \frac{a}{8kq^2}$

$$\Rightarrow \frac{a}{8kq^2} > 0$$

$\Rightarrow a > 0$ (which is assumed in the model).