Dynamic and Competitive Effects of Direct Mailings: A Charitable Giving Application

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Web Appendix

BAYESIAN ESTIMATION OF DIRECT MAILING RESPONSE MODEL PARAMETERS

We have N individuals with T_i mailing event observations in W_i weeks for individual i, i = 1,..,N. Define for mailing event τ $y^*_{i\tau} = (R^*_{i\tau}, A^*_{i\tau})^T$ and $\varepsilon_{i\tau} = (\varepsilon_{Ri\tau}, \varepsilon_{Ai\tau})^T$ and let $\lambda = (\lambda_m, \lambda_r, \lambda_a)^T$ contain all decay parameters. Let $X_{i\tau}(\lambda)$ denote the (1 x k)-matrix of k mean-centered explanatory variables, where λ in parentheses indicates the dependence on the decay parameters. Then $X_i(\lambda)$ is the (T_i x k) matrix that stacks $X_{i\tau}(\lambda)$ for the T_i mailing events of individual i. For y^*_i and ε_i similar definitions hold.

In our non-linear random-coefficients Tobit-II model specification in (1)-(4), we have $\varepsilon_{i\tau} \sim N(0,\Sigma_\varepsilon)$ with $\Sigma_\varepsilon = \begin{bmatrix} 1 & \sigma_{RA} \\ \sigma_{AR} & \sigma_A^2 \end{bmatrix} = \begin{bmatrix} 1 & \rho\sigma_A \\ \rho\sigma_A & \sigma_A^2 \end{bmatrix}$, $\beta_i \sim N(\beta,\Sigma_\beta)$ and $\beta_i = (\beta_{Ri}^T, \beta_{Ai}^T)^T$ of size (2k x 1). The vector β_{Ri} contains all parameters in the response equation, excluding the decay parameters, that is $\beta_{Ri} = (\beta_{R0i}, \beta^{own}_{R1i}, \beta^{other}_{R1i}, \beta^{own}_{R2i}, \beta^{other}_{R2i}, ..., \beta^{own}_{R7i}, \beta^{other}_{R7i})^T$.

To correct for target selection and the resulting endogeneity of the mailings received, we simultaneously estimate our Tobit-II model for the response decision and the mailing strategy models for all three firms. We assume that a firm makes a mailing decision every week w for each individual i. Let m_{ijw} be a dummy variable indicating whether firm j sends a mailing to individual i in week w, w = 1,…,W_i. Then m_i denotes the (W_ix3) matrix of mail dummies for all firms for all weeks of individual i. The mailing strategy model is a probit model for a firm’s weekly mailing decision as a function of the individual response parameters and week-dummies. Let Z_{1i} be the (W_i

x 2k) matrix that stacks W_i times the vector β_i^T . Furthermore, let Z_{2i} be a (W_i x 52) matrix containing an intercept and 51 week-dummies. Then we add the following to our model in (1)-(4):

(W1) $$P[m_{ijw} = 1 | Z_{1i}, Z_{2i}] = P[m^*_{ijw} > 0 | Z_{1i}, Z_{2i}]$$

(W2) $$m^*_i = Z_{1i}\delta_1 + Z_{2i}\delta_2 + \xi_i \text{ with } \xi_i \sim N(0,I)$$

Here, δ_1 is a (2k x 3) matrix containing for all three funds the parameters for the individual response parameters and δ_2 a (52 x 3) matrix containing the constant and the parameters for the week-dummies.

To obtain draws from the posterior distributions for the model parameters, we use the Gibbs sampling technique of Geman and Geman (1984). Furthermore, we make use of data augmentation (Tanner and Wong 1987) for the latent variables in the model. The latent variables y^*_i, β_i and m^*_i $\forall$i are sampled alongside the model parameters β, Σ_β, λ, Σ_ε, δ_1 and δ_2. We specify a flat prior for β and independent weakly informative priors for the other model parameters, details of which will be discussed below. Finally, when a full conditional posterior distribution is of unknown form we use the Metropolis-Hastings algorithm (Chib and Greenberg 1995). In the remainder of this appendix we describe for each parameter and each latent variable the full conditional distribution we use to obtain posterior results.

Sampling of y^*_i

To sample the elements of y^*_i, we use a data augmentation step and simulate the latent variables for each mailing event as follows. When a purchase is made, we set $A^*_{i\tau}$ equal to $A_{i\tau}$ and draw $R^*_{i\tau}$ from the conditional normal distribution $N\left(X_{i\tau}(\lambda)\beta_{Ri} + \rho\frac{(A^*_{i\tau} - X_{i\tau}(\lambda)\beta_{Ai}}{\sigma_A}, 1-\rho^2\right)$, truncated from below at zero. When no purchase is made, we start with drawing $R^*_{i\tau}$ from the conditional normal distribution $N(X_{i\tau}(\lambda)\beta_{Ri}, 1)$, truncated from above at zero. We then draw $A^*_{i\tau}$ from its conditional normal distribution $N\left(X_{i\tau}(\lambda)\beta_{Ai} + \sigma_A\rho(R^*_{i\tau} - X_{i\tau}(\lambda)\beta_{Ri}), (1-\rho^2)\sigma^2_A\right)$.

Sampling of m^*_i

To sample m^*_i, we use a data augmentation step by simulating the latent variables as follows. We draw m^*_i from the normal distribution $N(Z_{1i}\delta_1 + Z_{2i}\delta_2, I)$, with each element truncated from below at zero when a mailing is sent, or truncated from above at zero when no mailing is sent.

Sampling of β_i

As β_{Ri} and β_{Ai} are correlated, it is convenient to sample them simultaneously. For this purpose we define $Z_{i\tau}(\lambda) = I_2 \otimes X_{i\tau}(\lambda)$ with I_2 the 2-dimensional identity matrix and $\otimes$ the Kronecker product. Let $Z_i(\lambda)$ be the $(2T_i \times 2k)$ matrix that stacks the $Z_{i\tau}(\lambda)$ matrices for the T_i mailing events of individual i. Then $y^*_i = Z_i(\lambda)\beta_i + \varepsilon_i$ with $\varepsilon_i \sim N(0, I_{T_i} \otimes \Sigma_\varepsilon)$. In addition we have $\beta_i = \beta + \eta_i$ with $\eta_i \sim N(0, \Sigma_\beta)$. Finally, we have $m^*_i = Z_{1i}\delta_1 + Z_{2i}\delta_2 + \xi_i$ with $\xi_i \sim N(0, I)$ and $Z_{1i} = \iota \otimes \beta_i^T$.

Combining the three sources of information on β_i we obtain,

$\beta_i \mid y^*_i, m^*_i, Z_i(\lambda), \beta, \Sigma_\varepsilon, \Sigma_\beta, Z_{2i}, \delta_1, \delta_2 \sim N(VU, V)$ with $V^{-1} = Z^T_i(\lambda)(I_{T_i} \otimes \Sigma_\varepsilon)^{-1} Z_i(\lambda) + \Sigma^{-1}_\beta + \delta_1\delta_1^T \cdot W_i$

and $U = Z^T_i(\lambda)(I_{T_i} \otimes \Sigma_\varepsilon)^{-1} y^*_i + \Sigma^{-1}_\beta \beta + \sum_{t=1}^{W_i} \delta_1(m^*_{it} - Z_{2it}\delta_2)$ and a draw is made from this distribution.

Sampling of Σ_ε

Since $\Sigma_{\varepsilon 11}$ is restricted to 1 for identification purposes, sampling of Σ_ε is not straightforward. We follow the approach of McCulloch et al. (2000) and use the reparametrization

$\Sigma_\varepsilon = \begin{bmatrix} 1 & \gamma \\ \gamma & S+\gamma^2 \end{bmatrix}$ where S and γ are both scalars in our two-dimensional case. This implies

$\varepsilon_{Ri\tau} \sim N(0,1)$ and $\varepsilon_{Ai\tau} \mid \varepsilon_{Ri\tau}, \gamma, S \sim N(\varepsilon_{Ri\tau}\gamma, S)$. Now, consider $\varepsilon_{Ai\tau} = \varepsilon_{Ri\tau}\gamma + \omega_i$ and note that S is the variance of the error term in this model. Given conjugate priors $S \sim IG2(\kappa, C)$ and $\gamma \sim N(\bar{\gamma}, B^{-1})$, the full conditional posteriors are $S \sim IG2\left(\kappa + \sum_{i=1}^{N} T_i, C + \sum_{i=1}^{N}\sum_{\tau=1}^{T_i}(\varepsilon_{Ai\tau} - \varepsilon_{Ri\tau}\gamma)^2\right)$ and

$\gamma \sim N\left(A_\gamma\left(\frac{\sum_{i=1}^{N}\sum_{\tau=1}^{T_i}\varepsilon_{Ri\tau}\varepsilon_{Ai\tau}}{S} + B\bar{\gamma}\right), A_\gamma\right)$ with $A_\gamma = \left(\frac{\sum_{i=1}^{N}\sum_{\tau=1}^{T_i}\varepsilon_{Ri\tau}^{\;2}}{S} + B\right)^{-1}$.

We take $\bar{\gamma} = 0$, $B^{-1} = 1/10$, $\kappa = 3$ and $C = (1 - B^{-1})(\kappa - 1)$, in line with McCulloch et al. (2000) and draw S and γ from the full conditional posterior distributions.

Sampling of λ

To ensure that the effect of an event is diminishing over time the decay parameters must be in the interval (0, 1). To achieve this, we specify the decay parameter vector as:

(W3) $$\lambda = \frac{\exp(\varphi)}{1+\exp(\varphi)}$$

Thus, we apply the logit transformation to the vector λ to obtain a vector φ and generate draws for φ to ensure that the elements of λ are in the interval (0, 1). We use the Metropolis-Hastings algorithm (Chib and Greenberg, 1995) to make independent draws for the separate elements in φ and specify a univariate N(0, 1) prior distribution for each element φ_j, j = 1,…,J with J the number of elements of φ (see also Ansari, Mela and Neslin 2008). Then the full conditional posterior distribution for φ_j, j = 1,…,J is proportional to the likelihood times the prior and thus to

$$\prod_{i=1}^{N}\prod_{\tau=1}^{T_i}\exp\left(-\tfrac{1}{2}\left(y^*_{i\tau}-Z_{i\tau}\left(\frac{\exp(\varphi_j)}{1+\exp(\varphi_j)}\right)\beta_i\right)^T\Sigma_\varepsilon^{-1}\left(y^*_{i\tau}-Z_{i\tau}\left(\frac{\exp(\varphi_j)}{1+\exp(\varphi_j)}\right)\beta_i\right)\right)\cdot\exp\left(\frac{-\varphi_j^{\,2}}{2}\right)$$. We draw each element in φ sequentially using a random walk Metropolis-Hastings algorithm with a normal candidate-generating density centered on the previous draw. To obtain reasonable acceptance rates, the variance is adjusted depending on the acceptance rate (Train 2003, p.306).

Sampling of β

To sample β we consider the part of the model that depends on β which we can write as $\beta_i = \beta + \eta_i$ with $\eta_i \sim N(0, \Sigma_\beta)$. Given a conjugate prior $\beta \sim N(\overline{\beta}, B_\beta^{-1})$, β is drawn from

$N\left(A_\beta\left(\Sigma_\beta^{-1}\sum_i \beta_i + B_\beta\overline{\beta}\right), A_\beta\right)$ with $A_\beta = \left(N\Sigma_\beta^{-1} + B_\beta\right)^{-1}$. We take $\overline{\beta} = 0$, $B_\beta = 1/100$, and draw β from the full conditional posterior distribution.

Sampling of Σ_β

To sample Σ_β we again consider the regression model $\beta_i = \beta + \eta_i$ with $\eta_i \sim N(0,\Sigma_\beta)$. It follows that the full conditional posterior distribution of Σ_β is an inverted Wishart with scale parameter

$\sum_{i=1}^{N}(\beta_i - \beta)(\beta_i - \beta)^T + \kappa_1 I_{2k}$ and $N + \kappa_2$ degrees of freedom, where the κ terms stem from the conjugate prior we impose to improve convergence of the Gibbs sampler, as recommended by Hobert and Casella (1996). We set $\kappa_1 = 1/10$ and $\kappa_2 = 32$ to induce only a marginal influence of the prior on the posterior distribution and draw Σ_β from its full conditional posterior distribution.

Sampling of δ_1 and δ_2

To sample δ_1 and δ_2 we consider the regression model $m_i^* = Z_{1i}\delta_1 + Z_{2i}\delta_2 + \xi_i$ with $\xi_i \sim N(0,I)$. Let $Z_i = (Z_{1i}, Z_{2i})$ of size (W_i x (2k + 52)) and $\delta = (\delta_1^T, \delta_2^T)^T$ of size ((2k + 52) x 3). Given a conjugate prior $\delta \sim N(\bar{\delta}, B_\delta^{-1})$, δ is distributed as $N\left(A_\delta\left(Z_i^T m_i^* + B_\delta\bar{\delta}\right), A_\delta\right)$ with $A_\delta = \left(Z_i^T Z_i + B_\delta\right)^{-1}$. We take $\bar{\delta} =$ 0, $B_\delta = 1/10$, and draw δ from the full conditional posterior distribution.

ESTIMATION RESULTS OF MAILINGS STRATEGY MODELS

The tables below present the parameter estimates for the mailing strategy models. The first table presents the estimates of the parameters belonging to the parameters from the response equation of the Tobit-2 model, and the second table presents the estimates of the parameters belonging to the parameters from the amount equation of the Tobit-2 model. The third table presents the estimates for the constants and the week dummies in the mailing strategy model.

Table W1

PARAMETER ESTIMATES MAILING STRATEGY MODELS:

RESPONSE EQUATION VARIABLES

Explanatory variables	*Charity 1*		*Charity 2*		*Charity 3*	
β_0	.608**	(.197)	.561**	(.170)	.698**	(.240)
β_1^{own}	.389	(.288)	.021	(.223)	-.068	(.313)
β_1^{other}	-.039	(.305)	-.038	(.254)	-.075	(.366)
β_2^{own}	.101	(.303)	.004	(.283)	-.081	(.365)
β_2^{other}	.265	(.301)	.040	(.256)	.077	(.338)
β_3^{own}	.035	(.261)	-.129	(.214)	.276	(.302)
β_3^{other}	-.210	(.276)	.231	(.252)	-.459	(.330)
β_4^{own}	.098	(.254)	-.105	(.223)	.128	(.256)
β_4^{other}	-.126	(.318)	.094	(.253)	-.455	(.298)
β_5^{own}	.135	(.280)	.055	(.278)	-.007	(.307)
β_5^{other}	-.022	(.294)	-.095	(.285)	.179	(.329)
β_6^{own}	-.127	(.256)	-.549**	(.200)	-.025	(.287)
β_6^{other}	-.099	(.284)	.082	(.236)	-.068	(.336)
β_7^{own}	.065	(.292)	.068	(.255)	-.071	(.309)
β_7^{other}	-.267	(.281)	.198	(.267)	-.167	(.331)

**: Zero not contained in 95%, 99% Highest Posterior Density region, respectively.

Table W2

PARAMETER ESTIMATES MAILING STRATEGY MODELS:

AMOUNT EQUATION VARIABLES

Explanatory variables	*Charity 1*		*Charity 2*		*Charity 3*	
β_0	-.230	(.197)	-.145	(.172)	-.203	(.220)
β_1^{own}	.155	(.291)	-.032	(.283)	-.023	(.333)
β_1^{other}	.160	(.301)	-.048	(.268)	.149	(.342)
β_2^{own}	-.072	(.309)	-.020	(.272)	-.055	(.333)
β_2^{other}	.176	(.310)	-.034	(.267)	-.019	(.322)
β_3^{own}	-.156	(.292)	-.141	(.255)	.047	(.356)
β_3^{other}	.056	(.350)	.002	(.291)	-.045	(.384)
β_4^{own}	.061	(.222)	.046	(.185)	-.123	(.247)
β_4^{other}	-.008	(.295)	.040	(.260)	.050	(.343)
β_5^{own}	.011	(.300)	.058	(.284)	-.074	(.317)
β_5^{other}	.058	(.315)	.023	(.282)	.019	(.324)
β_6^{own}	-.207	(.257)	.202	(.185)	-.024	(.274)
β_6^{other}	-.199	(.290)	-.094	(.251)	-.268	(.327)
β_7^{own}	-.267	(.266)	-.233	(.255)	.235	(.324)
β_7^{other}	-.209	(.303)	-.114	(.268)	-.031	(.334)

Table W3

PARAMETER ESTIMATES MAILING STRATEGY MODELS:

CONSTANT AND WEEK DUMMMIES

Week	*Charity 1*		*Charity 2*		*Charity 3*	
constant	-.544*	(.254)	-.633**	(.233)	-.244	(.263)
1	-.815**	(.101)	-1.209**	(.070)	-1.607**	(.141)
2	-1.029**	(.124)	.178**	(.030)	-1.620**	(.146)
3	.173**	(.049)	.471**	(.027)	-1.610**	(.145)
4	1.876**	(.038)	-.359**	(.036)	-1.624**	(.144)
5	1.149**	(.040)	-.855**	(.050)	-1.620**	(.148)
6	-.194**	(.058)	1.083**	(.026)	-1.627**	(.148)
7	.396**	(.043)	.297**	(.027)	-1.630**	(.144)
8	1.041**	(.039)	.029	(.030)	-1.614**	(.141)
9	.737**	(.041)	-.309**	(.034)	-1.612**	(.144)
10	-.675**	(.083)	.021	(.030)	-1.626**	(.142)
11	-.984**	(.109)	.181**	(.028)	-.328**	(.058)
12	-.118*	(.054)	.330**	(.028)	1.113**	(.044)
13	-.349**	(.063)	.309**	(.028)	-.613**	(.065)
14	-.465**	(.069)	.234**	(.028)	-1.669**	(.136)
15	1.100**	(.039)	.501**	(.026)	-1.667**	(.139)
16	1.170**	(.039)	-.295**	(.033)	-1.567**	(.131)
17	.950**	(.039)	.935**	(.025)	-1.468**	(.113)
18	-.108	(.054)	.597**	(.026)	-.433**	(.057)
19	-1.188**	(.146)	-.147**	(.032)	.277**	(.048)
20	.628**	(.042)	-1.010**	(.054)	-1.157**	(.091)
21	.407**	(.044)	-.494**	(.037)	-1.677**	(.137)
22	-.187**	(.056)	.100**	(.029)	-1.617**	(.136)
23	.205**	(.046)	.697**	(.026)	-.238**	(.054)
24	-.556**	(.075)	.323**	(.027)	-.158**	(.052)
25	-1.041**	(.121)	.369**	(.027)	-.960**	(.076)
26	-.253**	(.059)	-.034	(.030)	.925**	(.044)
27	-.518**	(.069)	.324**	(.027)	-.796**	(.072)
28	.430**	(.043)	-.278**	(.033)	-1.687**	(.143)
29	-.240**	(.058)	.690**	(.026)	-1.188**	(.092)
30	-.660**	(.079)	-.611**	(.039)	-1.106**	(.088)
31	-.421**	(.064)	.193**	(.028)	-.624**	(.064)
32	-.709**	(.082)	.376**	(.027)	.100*	(.050)
33	.210**	(.046)	-.443**	(.037)	.034	(.051)
34	.776**	(.040)	-.520**	(.037)	-.732**	(.072)
35	.853**	(.039)	-.014	(.030)	-.496**	(.062)
36	.231**	(.046)	-.361**	(.034)	.008	(.053)
37	.097*	(.049)	-.314**	(.034)	-1.429**	(.119)
38	1.149**	(.039)	.150**	(.028)	-.394**	(.060)
39	.475**	(.044)	.771**	(.025)	-1.551**	(.137)
40	-.643**	(.080)	-.099**	(.031)	-1.014**	(.090)
41	-.537**	(.077)	.523**	(.027)	-.947**	(.083)
42	1.077**	(.039)	-.213**	(.032)	-1.135**	(.097)
43	-.161**	(.056)	-.492**	(.038)	1.305**	(.045)

44	.032	(.051)	.344**	(.028)	.164**	(.051)
45	.961**	(.040)	.338**	(.028)	-1.597**	(.145)
46	.526**	(.042)	.783**	(.026)	-1.603**	(.139)
47	1.757**	(.037)	.150**	(.029)	-1.588**	(.144)
48	.705**	(.041)	.952**	(.026)	-1.403**	(.123)
49	.745**	(.041)	.485**	(.027)	-1.503**	(.135)
50	.116*	(.052)	-.130**	(.032)	-.035	(.053)
51	-1.068**	(.132)	-1.667**	(.127)	.464**	(.049)

*, **: Zero not contained in 95%, 99% Highest Posterior Density region, respectively.

ESTIMATION RESULTS WITHOUT CORRECTION FOR ENDOGENEITY

The table below presents the parameter estimates for the model without the correction for potential endogeneity bias.

Table W4

PARAMETER ESTIMATES WITHOUT ENDOGENEITY CORRECTION

Explanatory variables		*Response equation*		*Amount equation*		*Decay*		
Constant	β_0	-1.257**	(.014)	1.229**	(.020)			
Mailings	β_1^{own}	-.266**	(.048)	-.142**	(.033)	λ_m	(3.56e-4)[a]	(1.65e-4)
	β_1^{other}	.065	(.052)	-.010	(.029)			
Mailings^2	β_2^{own}	-.050	(.034)	-.002	(.021)			
	β_2^{other}	-.032	(.028)	.003	(.016)			
Response	β_3^{own}	.443**	(.042)	-.053	(.049)	λ_r	.276[a]	(.015)
	β_3^{other}	.122	(.082)	-.188**	(.048)			
Response recency	β_4^{own}	.554**	(.055)	-.388**	(.048)			
	β_4^{other}	.252**	(.037)	-.221**	(.032)			
Response recency^2	β_5^{own}	-.005	(.016)	.025*	(.010)			
	β_5^{other}	-.008	(.015)	-.001	(.011)			
Amount	β_6^{own}	.228**	(.018)	.494**	(.022)	λ_a	.274[a]	(.014)
	β_6^{other}	.008	(.016)	.030**	(.012)			
Amount recency	β_7^{own}	-.248**	(.027)	.164**	(.021)			
	β_7^{other}	-.024	(.023)	.147**	(.016)			

*, **: Zero not contained in 95%, 99% Highest Posterior Density region, respectively.

[a]: Testing for significance is not relevant as implementation of the logit transformation automatically leads to exclusion of 0.

The table below presents the parameter estimates for the models with respectively charity 1 and 2, charity 1 and 3, and charity 2 and 3.

Table W5

PARAMETER ESTIMATES WITH TWO CHARITIES

Explanatory variables		*Response equation*		*Amount equation*		*Decay*		
Constant	β_0	-1.392**	(.024)	1.163**	(.023)			
		-1.036**	(.025)	1.346**	(.029)			
		-1.517**	(.025)	1.042**	(.031)			
Mailings	β_1^{own}	-.231**	(.050)	-.093*	(.048)	λ_m	.001[a]	(2.46e-4)
		-.193**	(.065)	-.062	(.054)		.002[a]	(.001)
		-.240**	(.033)	-.121**	(.053)		.001[a]	(1.69e-4)
	β_1^{other}	.153**	(.040)	-.067	(.042)			
		-.021	(.087)	.025	(.095)			
		.053	(.077)	-.006	(.051)			
Mailings^2	β_2^{own}	-.141**	(.037)	-.047	(.031)			
		-.150**	(.050)	-.059	(.039)			
		-.151**	(.026)	-.028	(.031)			
	β_2^{other}	-.084**	(.032)	.050*	(.024)			
		.017	(.098)	-.037	(.068)			
		.081**	(.040)	.012	(.033)			
Response	β_3^{own}	.300**	(.042)	-.064	(.041)	λ_r	.225[a]	(.010)
		.080	(.064)	-.088	(.058)		.373[a]	(.014)
		.455**	(.048)	.026	(.044)		.247[a]	(.010)
	β_3^{other}	.107**	(.050)	-.067	(.059)			
		-.051	(.151)	.017	(.089)			
		-.059	(.070)	-.017	(.099)			
Response recency	β_4^{own}	.369**	(.064)	-.509**	(.055)			
		.313**	(.077)	-.528**	(.056)			
		.298**	(.072)	-.348**	(.060)			
	β_4^{other}	.197**	(.048)	-.331**	(.076)			
		.033	(.076)	-.043	(.078)			
		-.132**	(.041)	.165**	(.055)			
Response recency^2	β_5^{own}	.002	(.030)	.027	(.020)			
		-.045	(.031)	-.004	(.019)			
		.025	(.026)	.003	(.025)			
	β_5^{other}	.045*	(.022)	.014	(.026)			
		-.004	(.069)	-.126	(.094)			
		-.019	(.041)	-.036	(.034)			

Amount	β_6^{own}	.095**	(.021)	.354**	(.021)	λ_a	.241[a]	(.009)
		.129**	(.031)	.483**	(.027)		.412[a]	(.019)
		.099**	(.024)	.541**	(.030)		.258[a]	(.011)
	β_6^{other}	.048*	(.022)	.044**	(.019)			
		.046	(.060)	-.019	(.053)			
		-.005	(.036)	.060	(.048)			
Amount recency	β_7^{own}	-.135**	(.038)	.225**	(.025)			
		-.014	(.040)	.277**	(.024)			
		-.070	(.043)	.191**	(.040)			
	β_7^{other}	-.064	(.043)	.174**	(.028)			
		-.107	(.081)	-.079	(.088)			
		-.038	(.086)	.025	(.037)			

*, **: Zero not contained in 95%, 99% Highest Posterior Density region, respectively.

[a]: Testing for significance is not relevant as implementation of the logit transformation automatically leads to exclusion of 0.

ESTIMATION RESULTS WITH ONE CHARITY (NO COMPETITIVE EFFECTS)

The table below presents the parameter estimates for the models with respectively charity 1, charity 2, and charity 3.

Table W6

PARAMETER ESTIMATES WITHOUT COMPETITIVE EFFECTS

Explanatory variables		*Response equation*		*Amount equation*		*Decay*		
Constant	β_0	-1.000**	(.032)	1.364**	(.032)			
		-1.605**	(.032)	.928**	(.039)			
		-.970**	(.046)	1.125**	(.043)			
Mailings	β_1^{own}	-.177**	(.064)	-.035	(.051)	λ_m	.007[a]	(.003)
		-.175**	(.048)	-.119*	(.055)		.005[a]	(.002)
		-.220**	(.079)	-.118*	(.074)		.022[a]	(.009)
Mailings^2	β_2^{own}	-.149**	(.042)	-.050	(.036)			
		-.138**	(.032)	-.050	(.033)			
		.013	(.044)	-.040	(.054)			
Response	β_3^{own}	.059	(.068)	-.047	(.061)	λ_r	.564[a]	(.026)
		.393**	(.064)	.032	(.084)		.344[a]	(.017)
		.141	(.107)	-.066	(.096)		.484[a]	(.037)
Response recency	β_4^{own}	.354**	(.049)	-.450**	(.052)			
		.249**	(.078)	-.419**	(.070)			
		.242**	(.060)	-.081	(.076)			
Response recency^2	β_5^{own}	-.044**	(.017)	-.026*	(.012)			
		.022	(.022)	-.004	(.019)			
		.060	(.037)	-.026	(.036)			
Amount	β_6^{own}	.072**	(.026)	.406**	(.031)	λ_a	.596[a]	(.028)
		.024	(.028)	.400**	(.036)		.336[a]	(.022)
		-.022	(.039)	.511**	(.063)		.489[a]	(.051)
Amount recency	β_7^{own}	-.001	(.020)	.265**	(.022)			
		-.012	(.041)	.270**	(.031)			
		.126*	(.046)	.185**	(.043)			

*, **: Zero not contained in 95%, 99% Highest Posterior Density region, respectively.

[a]: Testing for significance is not relevant as implementation of the logit transformation automatically leads to exclusion of 0.